OPTIMUM DESIGN OF ELECTRICAL MOTORS FOR MULTILINK MANIPULATORS AND AUTOMATED MANUFACTURING SYSTEMS

BY

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A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN
PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

1984

ACKNOWLEDGMENTS

The author gratefully acknowledges the help of his advisor, Dr. A.M. Meystel and coadvisor, Dr. J.K. Watson, for the guidance, encouragement, and invaluable professional support.

The author also thanks the members of his committee, Dr. D.I. Elgerd, Dr. R.L. Sullivan, and Dr. D. Tesar, for the helpful discussions and positive contributions they have made.

Computing was done utilizing the facilities of the Northeast Regional Data Center of the State University System of Florida and Harris computer system, both of which are located in the University of Florida in Gainesville.

This work was supported by the Electrical Engineering Department at the University of Florida.

TABLE OF CONTENTS

PAGE

ACKNOWLEDGMENTS ii

ABSTRACT vi

CHAPTER

I INTRODUCTION 1

- 1.1 Problem of Optimum Design of Electrical Motors 1
- 1.2 State of the Art in Electrical Motor Design 8
  - 1.2.1 Conventional Design 12
  - 1.2.2 Initial Stage of Computer Use 15
  - 1.2.3 Early State of Optimum Design 16
  - 1.2.4 Optimum Design 17

II ANALYSIS OF DATABASE "MOTOR-USA" 22

- 2.1 Structure of Data 24
- 2.2 Analysis 26
  - 2.2.1 The Characteristic Parameters 26
  - 2.2.2 Comparison of T/W Ratio 27
  - 2.2.3 Comparison by Manufacturing Company 29
  - 2.2.4 Characteristic Parameters for DC PM Motors 38
- 2.3 Conclusions 38

III GENERAL APPROACH TO THE PROBLEM OF OPTIMUM DESIGN OF AN ELECTRIC MOTOR 44

- 3.1 Types of Optimum Design 44
- 3.2 General Problem of Electrical Machine Optimization 46
  - 3.2.1 Modelling and Formulation of Magnetic System 46
  - 3.2.2 Design 51
  - 3.2.3 Optimization Strategy 53
- 3.3 Transformer Design 58
  - 3.3.1 Perparation for Optimization 58
  - 3.3.2 Optimization 58
  - 3.3.3 Result 61

3.4 Synchronous Motor with PM Rotor 61
  3.4.1 Preparation for Optimization 61
  3.4.2 Optimization 61
  3.4.3 Result 61
3.5 Conclusions 68

IV STRUCTURE OF DESIGN PROVIDING MINIMUM WEIGHT/LOSSES OF ENERGY 69

4.1 The Specific Characteristics of DC PM Motors 69
4.2 Comparison of Permanent Magnet Materials 76
4.3 Design 80
  4.3.1 Normal Structure of Motor 85
  4.3.2 "Inside-out" Structure 99
4.4 Optimization 101
  4.4.1 Computer Program 103
4.5 Result of Optimization 104
  4.5.1 Convergence Check 104
  4.5.2 Computer Output 106
  4.5.3 Numerical Result with Different Torques 108
  4.5.4 Comparison with Different Material and Structure 110
4.6 Conclusions 110

V DESIGN OF DC PM MOTOR FOR THE ENHANCED TIME OF OPERATION 113

5.1 Formulation of Operation Time Enhancement 116
  5.1.1 Objective Function 116
  5.1.2 Bending Capacity of Spur Gear Teeth 123
5.2 Time Reduction 126
  5.2.1 Functional Relationship 126
  5.2.2 Computation of T_r and g_r 128
  5.2.3 Motor Performance Parameters 134
5.3 Numerical Example 135
5.4 Conclusions 135

VI MOTOR DESIGN STRUCTURES, CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH 138

6.1 Design Structure 138
6.2 Conclusions 141
6.3 Suggestions for Future Research 144

APPENDICES

A TRANSFORMER 145

B SYNCHRONOUS MOTOR WITH PM ROTOR 148

C FLOW CHART OF COMPUTER PROGRAM 156

D FLOW CHART OF COMPUTER PROGRAMS 169

REFERENCES 204

BIOGRAPHICAL SKETCH 209

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of the University of Florida in Partial Fulfillment of the
Requirements for the Degree of Doctor of Philosophy

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August 1984

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This dissertation addresses the problem of finding the optimum design of electric motors used as actuators in robotic manipulators and in manufacturing machines. The optimization may provide a minimum time of operation and/or minimum losses of energy for the actuator.

The development of design technologies is reviewed, yielding a recommendation for a hierarchical procedure for motor design using small computers.

Information from the catalogues of 1800 motors was organized into a computer database, "MOTOR-USA." Analysis of the database shows that the DC PM motor has the best performance characteristics among all types of motors. Further analysis reveals the possibility of the existence of a unique motor configuration, different from existing configurations, and with better performance characteristics.

The design of a generalized electric motor is formulated using a doubly excited magnetic field system. The generalized examples of transformer and synchronous motor designs lead to an important result: there exists an optimum machine configuration which remains unchanged when the power of the motor is changed over a broad range.

The design of a DC PM motor is optimized for minimum weight, or losses, or a combination of both objective functions. The design procedure is developed, and results are compared for two different materials (ferrite vs. rare earth PM) and for two different structures (normal vs "inside-out"). The comparison shows the DC PM motor with a rare earth magnet in an "inside-out" structure with a definite "length/diameter" ratio to be a more desirable solution for a robotic application because the weight and inertia are reduced noticeably under the condition of reduced time of operation.

The time of operation is reduced using the hierarchical design procedure and using the constraint of maximum bending stress of the gear teeth. The time of operation is reduced by matching load and actuator.

The results of the research help to improve the method of electric motor design as well as the performance of the mechanical equipment which is driven by the motor.

CHAPTER I
INTRODUCTION

1.1 Problem of Optimum Design of Electric Motors

Since the possibility of obtaining mechanical motion using electromagnetic means was discovered by Faraday in 1821 [1], electric motors have become the most versatile and efficient devices for electromechanical energy conversion. Thus, electric motors are being used in all areas, and design techniques have been developed by manufacturers to improve the performance characteristics while maintaining the specifications of the motors.

The performance of a motor can be determined by its efficiency [2-4], acceleration [5], speed-torque characteristics, and the temperature of the winding in a variety of work-regimes. Most of these parameters are related to the weight and losses of a motor. Another way of judging the motor performance at the stage of motor selection among many motors is to compare the modified parameters, such as torque per weight (T/W) [6] or power per weight (P/W) [7-11]. These parameters can better characterize the energy conversion capacities of the motor via the values of torque or power to be produced by the unit weight of the motor. The specifications of the motor are represented by output power, operating voltage, and rated speed.

The word "design" means selecting principles of operation, geometry, dimensions, and materials which can satisfy the specifications and the performance requirements. There is an infinite number of possible design solutions. The optimum design is determined as the most economical and most suitable solution among the multiplicity of admissible designs. The criteria in the optimum design or in the selection of a motor from the list of available choices can be cost, weight, or losses of the motor combined with the satisfaction of the requirements of the work-regimes (such as the temperature or the time of operation). Each criterion can be used as a cost function or an objective function in optimum design.

The approaches to the design procedure changed while the status of motor applications changed over time and became more diversified in its purpose. In the 1920s and 30s, one motor gave mechanical power to several load devices using a belt as a power transmission device. All of the machines were driven by one motor, and the load on this motor averaged the individual load cycles of different machines. Therefore, at this time each load was not to be specified. In the 1930s and 40s, a single motor started to be used to handle a single machine, so users began to individualize the load requirements. There were still many different mechanical operations; so it was necessary to average the load with time. The ideas of "one-hour-load" and so on appeared in the practice of the design, simplifying the process of dealing with a stochastic load on the shaft. Beginning in the 1950s, several motors were used to operate a single machine or mechanical system, and the load conditions could be determined precisely by individualizing each motion of this system. The ultimate example of this system is an industrial robot where sometimes six to ten motors are used in the robot, and each motor characterizes each motion of the system. In this way, the applications of motors have been specialized to improve the motor performance by matching each load and motor condition.

The motor design can be grouped into two types (as shown in Fig. 1-1), which correspond to the status of knowledge of the individual customer and his requirements:

TYPE I: Individual customer is unknown so the general requirements from a group of similar customers will be formulated.

TYPE IIa: Individual customer is known, but the machine to be equipped by a motor is not designed yet.

TYPE IIb: Individual customer is known, and the machine to be equipped by a motor is already designed.

In TYPE I design, the motor will be independent of the customer's requirements. The generalized requirements, listed below, will be used as input requirements.

1. The average load torque on the shaft and its variation.

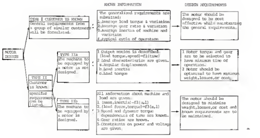

KNOWN INFORMATION
DESIGN REQUIREMENTS
MOTOR DESIGN
TYPE I CUSTOMER IS KNOWN
General requirements from a group of similar customers will be formulated.
TYPE II
Customer is known
specific requirements can be listed.
TYPE IIa
The machine to be equipped by a motor is not designed.
TYPE IIb
The machine to be equipped by a motor is designed.
The generalized requirements are submitted:
1 Average load torque & variation
2 Average gear ratio & variation
3 Average inertia of machine and variation
4 typical cycle of operation
1 Output motion is described (load torque,speed)=f1(time)
2 Load characterisrics are given.
a Angular displacement
b Load inertia
c Load torque
All information about machine and load are given.
1 (mass,inertia)=f2(α,l)
2 (load force,torque)=f3(α,l)
3 Speed and dynamic torque dependencies of time are known.
4 Gear ratios are known.
5 Constraints on power and voltage are given.
The motor should be designed to be cost effective while maintaining the general requirements.
1 Motor torque and gear are to be selected to have minimum time of operation.
2 Motor should be optimized to have minimum weight,losses,or cost.
The motor should be designed to minimize weight,losses,or cost and known requirements are to be maintained.

Fig.1-1 Two types of motor design

2. The average gear ratio in the machine and its variation.
3. The average inertia of the machine and its variation.
4. The typical cycles of operation.

Under all these conditions, the motor should be designed to be cost effective while maintaining the above requirements.

In TYPE IIa design, only the output motion of the machine to be designed is described, and the load characteristics are given as follows:

1. (Load torque, speed) = f (time)
2. Displacement
3. Velocity trajectory
4. Load inertia

In this case, the motor torque and gear ratio are selected to provide the minimum time of operation. After this, the motor should be designed optimally to minimize weight, losses, or cost of motor under the obtained motor torque.

In TYPE IIb design, all the mechanical information about the machine can be obtained:

1. (Load masses, inertia) = f (a, 1)
2. (Load forces, torque) = f (α, 1)
3. Speed and dynamic torque dependencies on time are known
4. Gear ratios are known

This motor should be designed to minimize weight, losses, or cost, and the above conditions are to be maintained.

This study will concentrate on the motor design for actuators in robotic manipulators and automated manufacturing systems. Since the robots of today employ almost every kind of electric motor, the technical and economic characteristics of robot applications produce a clear trend toward higher performance drives.

The ideal actuators for manipulators must provide high productivity, low initial cost, accuracy, uniformity of motion, low losses, and high reliability. Actually, it is not possible to achieve the best values of all of these properties, as each superior characteristic requires a compromise in the design of robotic actuators. The economics of robot application demand that a robot have a sufficient productivity to pay for itself in a short time. The actuator systems in present manipulators have the following inherent problems:

1. 80-90% of work is for carrying its own weight.
2. 40-50% of the weight is in actuators, gearboxes and supplemental equipment.
3. There is a very wide range of dynamics.
4. The main factors of errors are
    a. Backlash of gears.
    b. Losses of energy in the actuator increases the temperature of the motor structure and dimensional expansion will cause performance errors.
    c. The stiffness of the actuator system will have positional errors which are different from the load.

The theory of electric motor optimum design can be tested in application to the optimum design of the actuator for a multilink manipulator to solve the above problems. Most of the conditions that this actuator is to satisfy are individual, which gives us an opportunity to compare the solution with several conventional alternatives over a variety of performance indices to be achieved.

The following technical requirements can be set:

1. The weight of the motor is to be minimized [11-12].
2. The motor is to provide a variety of operational cycles with output path, speed, and acceleration, and jerk restricted.
3. The time of these cycles should be minimized.
4. The motor should not be a source of heat in construction which will destroy the accuracy of the machine.
5. Actuator volume can be a part of link structure volume and excessive volume reduces dexterity of a manipualtor.

These requirements reflect the present state of the art in the design of electric motor drives for robotics.

The study of actuating systems can be divided into two areas: matching the load and actuator, and optimizing the velocity trajectory using proper design of a control system. The control theory has been improved to upgrade the manipulator performance. Luh [13] analysed the motion control for industrial robots to have minimum positioning error. By controlling the robot at the joint level, on-line coordinate transformations are eliminated to reduce the computing time. Paul [14] formulated the dynamics equation and solved the control of manipulators so that the specified position trajectory can be achieved by adjusting the feedback gain.

However, the design of the electric motor for the actuator is still separated from the design of such system components as the converter, gearbox, and other components of the control system. To upgrade the whole system, a study to match load and actuator systems should be pre-executed before the design of control systems. This matching load and actuator system can maximize the performance indices by optimization.

1.2 State of the Art in Electric Motor Design

Although the design technology of the motor has been improved, there are still numerous design techniques in different manufacturing companies because the design technology developed by each company is kept secret from the others. For example, in the Gould Electric Company's report [15], the computer aided design procedure of the induction motor tells just the method of making the input list to the computer program. (The major portion of this method is based on Alger's book [16].) All the papers related to the design algorithm of motors published in the last 20 years are from individual scientists rather than from manufacturing companies.

Before explaining each design method, let us first discuss the general procedure of optimum design. When the motor is to be designed or selected, the type of motor application should be clarified as show in Fig. 1-1. The known information is different from design types as listed in Table 1-1. Normally, the load duty cycles are in complicated form; so an average of load cycles is taken to determine the rated torque or power. When all the specifications based upon Table 1-1 are considered at the initial stage of design, the dependencies between all optimum design methods will be set as follows to organize the design structure in a hierarchical manner:

1. The order of the design heirarchy is decided by the importance of the design criteria which are stated by the customer and by their involvement in different design.
2. The mandatory part of motor specifications are considered first, and the rest will be applied in the next stage of the design.

Based on the type of motor design in Fig. 1-1, the structure of an optimum design procedure is organized as shown in Fig. 1-2. This figure shows an example of TYPE I motor design. From the main part of input specifications and the initial values of design variables, the basic motor structure is determined. The design computations will find parameters which are necessary to compute the major criteria. The geometrical constraints, which are to be kept strictly,

Table 1-1 Initial design information

	TYPE I DESIGN	TYPE IIa DESIGN	TYPE IIb DESIGN
LOAD (MACHINE) INFORMATION	Generalized requirements 1.Average load torque and its variation 2.Average gear ratio and its variation 3.Average load inertia and its variation 4.Typical cycle of operation	1 Output motion (load torque,speed)=f1(time) 2.Load characteristics a Displacement b.Load inertia 3 Accuracy requirement	1 All machine information a (mass,inertia)=f2(α,l) b.(load dorce,torque)=f3(α,l c.Speed and dynamic torque dependencies on time d.Gear ratio 2 Load characteristics a.Displacement b Load inertia c Load torque 3 Accuracy requirement
POWER SUPPLY	Common power supply	should be selected	should be selected
DESIGN CRITERIA	1.Minimum cost	1 Minimum operating time 2.Minimum weight,losses, or cost	1.Minimum weight,losses, or cost

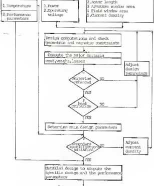

INPUT SPECIFICATIONS
SECONDARY SPEC.
1.Temperature
2.Performance parameters
MAIN SPECIFICATIONS
1.Power
2.Operating voltage
INITIAL VALUES OF DESIGN VAR
1.Rotor diameter
2.Rotor length
3 Armature window area
4 Field window area
5.Current density
Design computations and check geometric and magnetic constraints
Compute the major criteria
cost,weight,losses
Adjust design parameters
? criterion enhanced
NO
YES
? best criterion
NO
YES
Determine main design parameters
? secondary specifications satisfied
NO
Adjust current density
YES
Detailed design to compute the specific design and the performance parameters
FINAL DESIGN

Fig. 1-2 Optimum hierarchical design structure
(TYPE I DESIGN)

and the magnetic constraints will be checked. If these constraints are satisfied, the design is compared with others by the major criterion which is the cost function of an optimum design. If it is not the optimum design, another design will be tried by adjusting the design parameters until the optimum is found. This step of finding the basic structure is called the first step optimization, the second step optimization will find the detailed design parameters for the specific design and the performance check.

The stages development of design technology can be arranged as follows:

1. Conventional design
2. Initial stage of computer use
3. Early stage of optimum design
4. Optimum design

1.2.1 Conventional Design

At the beginning of this century, a motor was designed by the trial-and-error method [6, 16, 17], when the proper values of motor dimensions, number of windings and the voltage level were given, the performance of the assembled motor was checked and modifications tried by some variations of the design parameters from experience.

Figure 1-3 illustrates the conventional design of the DC motor. From the input parameters--power, voltage, and speed--the main dimensions of rotor, diameter and length of rotor are determined by the information of similar motors.

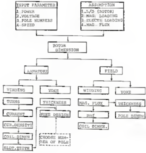

INPUT PARAMETER
1 POWER
2.VOLTAGE
3 POLE NUMBERS
4 SPEED
ASSUMPTION
1.L/D (ROTOR)
2 MAG. LOADING
3 ELECTR LOADING
4.MAG. FLUX
ROTOR DIMENSION
ARMATURE
FIELD
WINDING
YOKE
TURNS
THICKNESS
CURRENT
YOKE DESIGN
CUR.DENSITY
COIL DIMEN
CHOOSE NUMBER OF POLE
SLOT,TEETH
WINDING
YOKE
MAG. FLUX
THICKNESS
mmf
POLE DIMEN
COIL DIMEN.

Fig. 1-3 Conventional design structure of a DC motor

Then the magnetic flux density and magnetomotive force (mmf) are determined in the same way. In armature design, the number of armature windings and current are computed from the operating voltage, speed, and magnetic flux. After the number of slots per pole is given, the armature yoke dimension is computed. To generate the required magnetic flux, the mmf of field winding is computed using the magnetic circuit theory. Then the diameter of field coil and field pole dimensions are computed successively. After all these detailed dimensions of motor are determined, the losses and weight of each part are obtained.

In this design, the experience translated into a number of heuristic rules is applied to make minor changes which may improve the design or correct it to specific requirements. Thus, a good mathematical model of the motor design is not required.

However, the output of a motor depends on the dimensions of rotor, mmf of winding, and the speed of the motor. The values of these parameters must, therefore, be chosen to have optimum cost function, such as weight, manufacturing cost, or losses under a set of operating constraints. Since the mmf of winding can be expressed as a function of rotor dimensions and current density, these rotor dimensions become the most important factors in deciding the cost function or performance of a motor.

In conventional design, these parameters are determined by choosing suitable values obtained from similar motors or from experience, but these parameters can be determined from the optimization which may be different for a different purpose or cost function. When all of these variables are computed from the information of similar motors or from experience, the final result will not be the optimum design.

1.2.2 Initial Stage of Computer Use

When the computer was first introduced, the majority of the earliest problems to be attempted in the field of motor design by computer were of the formula-translation types. Many of these problems and their solutions have been described in relation to electrical motors. In these cases, the computer served as an aid to the designer by allowing a large number of calculations to be made and more accurate methods to be used and by disposing of the routine and tedious calculations.

Several studies [18-26] are categorized in this type of design. At this stage, they translated the formulas of motor design parameters into computer languages. Based upon the designer's insight and previous experience, a set of dimensions and parameters can be chosen, and if the constraints are not satisfied, the designer must draw upon his insight to modify the parameters to yield a motor whose performance will meet the specifications. These whole sequences which are similar to those of conventional design are repeated under different values of design parameters to compare the result. Actually, there were no algorithmic improvements in optimization of motor design.

1.2.3 Early Stage of Optimum Design

As the computer becomes faster, the algorithm of finding the minimum point of cost function using numerical methods improves. In 1965, Erlicki and Appelbaum [27] found the minimum overall cost of an induction motor by an algorithm they developed. This method consisted of assuming numerical values for the (n - 1) variables and determining the minimum for the n^{th} variable. This value is substituted in the equation, and the minimum is found for another variable at (n - 1) constant numerical values for the rest, including the minimum point found earlier. This procedure is repeated until all n variables are found. But this design problem is treated as one with unconstrained minimization; hence, the results do not correspond to reality.

Anderson [28] used Monte Carlo routines to find the minimum cost of a hydroelectric generator. After computing the material cost plus the capitalized cost of losses at a certain starting point, variables on all design parameters are checked to find the cost decreasing directions. If the tries of some variables are successful in reducing cost, the movement continued in that direction; if not successful, opposite directions are tried. These processes will be continued until the minimum point is found. Other studies [29, 30] are similar to those two approaches. The summarized description of each design is shown in Table 1-2.

These methods were the first tries of optimization before the nonlinear programming algorithm was widely developed. Those algorithms took more computing time because of slow convergence and sometimes would not converge to an exact optimum point because of the discontinuity of cost functions and constraints. When the systematic search to the optimum point is employed by finding the effective descent direction which was based upon the nonlinear programming algorithm, the global optimum will be achieved. But this clearly showed the direction in which an optimum design would proceed.

1.2.4 Optimum Design

As optimization theory and the search algorithm were more widely developed, the convergence speed was increased by searching effectively the cost function through the decreasing direction. As a result, the computing time is reduced, and a global optimum point is obtained.

Schinzinger [31] formulated the transformer design as a problem in nonlinear programming. The cost function of a transformer, which is the sum of material cost and operating cost, is minimized subject to constraints such as efficiency and temperature rise. He used the univariate search which finds the descent direction by changing a single variable at a time and moves to the descent direction to further reduce the cost function.

Table 1-2 Summary of each design method

REF	AUTHOR	MACHINE	POWER / SPEED(RPM)	COST FUNCTION	SEARCH METHOD	RESULT	APPLICATIONS
27	Erlicki	induction motor	/ 1800	material cost operating cost	find minimum values in each variables	unconstrained optimum (not realizable)	
28	Anderson	hydroelectric gen		material cost operating cost	find cost decreasing direction	not exactly a global minimum	
29	Godwin	induction motor		material cost	random search	not exactly a global minimum	
30	Chalmers	induction motor		losses	gradient method	not efficient method	
31	Schinzlinger	transformer	1000 VA /	material cost operating cost	univariate method	global minimum	optimization algorithm
32,33	Ramarathnam	induction motor	13.4 HP / 1800	material cost	gradient method & density function	global optimum	comparison of each optimization
34	Menzies	induction motor	884 HP / 1800	material cost	pattern search p^th optimization method	global optimum	
35	Fulton	induction motor	4.5 HP / 1500	material cost	intuitive comparison using graph	not exactly a global minimum	simplified optimization
36	Feras	synchronous motor	26.8 HP / 56000	minimum size	pattern search, p^th optimization method	global optimum	high speed motor design
37	Campbell	DC PM motor	7.5 HP / 1100	weight	intuitive comparison simplified analysis	not exactly a global minimum	rare earth PM motor design
38	Ramamoorty	reluctance motor	2 HP / 1500	material cost	Powell's unconstrained optimization	global minimum	
39	Lichtenstein			production cost	SUMT boundary search p^th optimization	global minimum	
40	Yuan	inductor		weight	Lagrange multiplier method	global optimum	

Ramarathnam and Desai [32, 33] used Sequential Unconstrained Minimization Technique (SUMT) to optimize the design of the induction motor with respect to the material cost. They used external penalty function to change a constrained problem into a unconstrained one, and a first-order gradient method was employed to move into descent direction.

Menzies and Neal [34] formulated the optimization problem for large induction motor design using the pattern search and penalty function method. The objective function is the sum of material cost and a specified constrained cost which is related to constraints.

Fulton et al. [35] found the optimum design in a different way from the other studies listed above. They checked the variations of the material cost of an induction motor graphically with three variables and found better design with less cost. The more economical combination of core stack length, wire diameter, and number of turns per coil could be found by repeating these procedures. This method is simple, but the exact optimum value is hard to find by this method.

Perez and Kassakian [36] designed a high-speed synchronous motor with respect to minimum size using the pattern search method of Hooke and Jeeves and an objective function which is a variant, proposed by Menzies and Neal [34], of the least p optimization. This allows the search to start from nonfeasible regions.

The axial field DC PM motor was designed by Campbell et al. [37]. After the initial analysis of finding the ratio of inner and outer diameter of the motor, the computer-aided design was applied to find the final optimum design. In this analysis step, assumptions were made in order to make the analytic computation possible, but the computer could have solved the problem accurately without any assumption. There were some other studies [38-40] which were similar to those approaches mentioned above.

As can be seen from all these optimization strategies shown in Table 1-2, the algorithms of finding the optimum were improved to search for the global optimum with less computing time. The targets of these designs were the general purpose motors that satisfy the requirements from broad market customers. In most of the optimum designs, the cost function or objective function is formulated using many variables and optimized with respect to all design parameters at one time. This may make the optimization problem more complicated.

This study divides the whole optimization procedure into several design stages, and each design stage is optimized and linked together into one software. The number of design variables is reduced by expressing some parameters with design variables. As a result, this hierarchical design, with fewer variables, can noticeably simplify the design. This kind of hierarchical design has the advantage of flexibility; a detailed design can be done using the same main software program.

This dissertation is organized as follows. In Chapter I, the strategies of motor design are reviewed, and two types of motor design are defined. In Chapter II, a database is organized, and the characteristic parameters of motors are compared for different types and for different manufacturing companies. In Chapter III, the generalized electric motor is formulated, and transformer and synchronous motor designs are optimized. In Chapter IV, a DC PM motor is optimized, and the result is compared for two different materials and for two different structures. In Chapter V, the design of DC PM motor for multilink manipulator is carried out to reduce the time of operation using the constraint of maximum bending capacity. In Chapter VI, this dissertation is concluded with suggested design structure, conclusions, and suggestions for future research.

CHAPTER II
ANALYSIS OF DATABASE 'MOTOR-USA'

All of the industrial processes require the electromechanical energy conversion from a readily available form to another form more suitable for utilization. There is, thus, a large variety of modifications of electric motors used in different branches of industry. The real expressions of those electromechanical models are reflected in sets of data for all the products that absorb multi-dimensional purposes. The information that may be presented for the customers is concentrated in the catalogues. From these catalogues, one can extract the representative characteristics of those motors. Different companies submit different representative characteristics in their catalogues. Therefore, the lacking data should be evaluated analytically or are subject to a special inquiry.

An attempt is undertaken to collect, within the framework of a single database, the representative characteristics for a large number of motors (manufactured in USA). The database is described in this study. It is organized on the basis of the collected catalogues for the following purposes.

1. To find out the state of the art in the motor manufacturing industry by computing the

characteristic parameters designating the correlations between T/W and between power, and efficiency and power.

2. To provide the information on motor selection by comparing the characteristic parameters of different types, different speeds and powers.
3. To find out whether the optimum geometry of the motor is actually present in industry.

This database is an integration of the distinct data of all types of electric motors and is stored in NERDC disks. The access to this database is handled by SAS software which is a computer system for data analysis and provides the following tools needed for data analysis [41]:

1. Information storage and retrieval
2. Data modification and programming
3. Statistical analysis
4. File handling

So this database has the following advantages among the general characteristics of database [42] in the analysis of motor data:

1. Redundancy can be reduced by selecting independent data entries.
2. The data can be shared among different users by using different application programs in the SAS software.
3. The information in this database can be easily updated.

4. The number of words in the list of characteristic parameters can be easily increased.

2.1 Structure of Data

During the preparation of the database, more than 300 manufacturing companies were screened, and 1815 motors from 34 manufacturing companies were selected based upon the following selection criteria

1. The general purpose motors, except gear motor, torque motor, and servo motor, are chosen as the target of analysis
2. The information for weight, power, and speed must be described in the catalogue.

The information in this database includes power, voltage, current, dimensions, efficiency, weight in AC motors And DC motors add inertia of motors to the information of AC motors.

The power ranges of motor application in common use are divided into five subgroups, as shown below, because some types of motors have concentrated in certain power ranges as shown in Fig. 2-1.

power group C1 = 0.0001 - 0.1 HP
C2 = 0.1 - 1.0 HP
C3 = 1.0 - 5.0 HP
C4 = 5.0 - 20.0 HP
C5 = 20.0 - 200.0 HP

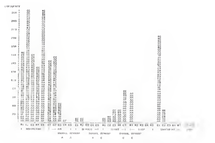


Fig. 2-1 Number of motors in each type, power, and RPM group

The RPM groups are divided into three subgroups because in AC machines the speed ranges in America are 1200, 1800, and 3600 RPM.

RPM group A = 0-1200 RPM

B = 1200-1800 RPM

C = 1800-3600 RPM

The characters 'A'-'E' are the power groups of C1-C5, respectively. From this figure, one can determine the power ranges of major applications in each type of motor because the number of catalogues can explain the number of motors manufactured and used in common use. For example, induction motors are used in all power ranges while synchronous motor and DC series motors are used in small power ranges for general purposes. In DC motors, series and shunt motors are being replaced by permanent magnet motors, and the development of rare earth PM material will accelerate this replacement to the high power range applications.

2.2 Analysis

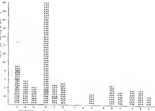
2.2.1 The Characteristic Parameters

The performance of a motor can be judged and compared with other motors by checking the specifications and comparison criteria as was shown in Table 1-1. The reference for motor evaluation in comparison with other motors can be set by several characteristic parameters. For the selection of motors in actuators for robot, aircraft, and automatic manufacturing systems, higher power, torque, and efficiency with less weight are the primary criteria.

For that purpose, power per weight (P/W) can be used, which expresses the power capacity that can be transformed from electrical energy into mechanical energy. In the same way, torque per weight (T/W) presents the torque capacity of a motor per unit weight. This T/W ratio is similar to the output coefficient [6, 16, 17] that has been used for a long time as a first step in designing electric motors. The coefficient is defined as the output power per unit speed and unit volume. Since the weight of a motor is proportional to its volume, the output coefficient is proportional to its T/W ratio. The T/W ratio is more meaningful than P/W ratio because the torque of a motor, rather than its power, decides the size of the motor. Efficiency can be used as a characteristic parameter of motor because it is related to temperature rise and rating of motors.

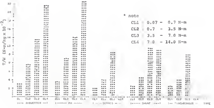
2.2.2 Comparison of T/W Ratio

From the data, the T/W ratio can be compared in each subgroup. In a very small power group C1 (0.00001-0.1 HP), the bar chart in Fig. 2-2a shows the average values of T/W in each type of motor by different RPM groups. DC PM motor shows the highest ratio in RPM group A because PM motors can be applied at very low speed. The induction motor also shows higher ratio because of its simple structure. When the motor is sorted by torque ranges, the PM motor shows the highest



a. Power class = C1 (0.0001 - 0.1 HP)

Fig 2-2 Comparison of T/W ratio in each type



b. Different torque range

Fig. 2-2 - continued

ratio than any other types of motors, as shown in Fig. 2-2b. This fact shows that the PM motor has better characteristics in small power range with ferrite and alnico materials, but when rare earth PM material is used, the difference between PM motors and other types of motors will become larger.

Figure 2-3 illustrates the dependencies between the average value of T/W ratio and the rated power for induction motors and DC motors. The plain line shows the best values, and the dotted line shows the average values of the ratio. From these curves, the approximated formulas can be determined as shown in Table 2-1. We can use these formulas as references at the initial stage of motor selection. For example, if the power range and speed are known, then the best and mean values of T/W ratios and weights of motors manufactured in industry can be computed successively (similar attempts are known from [7]). The detailed values of T/W ratios in each type, speed, range, and power are given in Table 2.2. From these data, one can get more detailed information on the weight and torque of motor that will be used.

2.2.3 Comparison by Manufacturing Company

According to the collected data, the specifications of motors are substantially dispersed, which reflects not only the variety of applicational requirements but also some differences in accepted approaches to a design procedure. The trade-off between determination to provide a number of

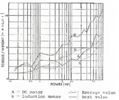


Fig. 2-3 T/W ratio vs. power

Table 2-1 Approximations of $P/W=f(P)$ and $T/W=f'(P)$ for different types of motors

TYPE	P/W vs. P	T/W vs. P
DC MOTOR	$P/W = 0.060P^{0.15}$	$T/W = 0.305P^{0.15}$
INDUCTION MOTOR	$P/W = 0.071P^{0.32}$	$T/W = 0.264P^{0.33}$
SYNCHRONOUS MOTOR	$P/W = 0.084P^{0.54}$	$T/W = 0.157P^{0.30}$

Table 2-2 T/W ratios for motors in the range of rated powers ≤ 100 HP

Type		Speed ≤ 1000 RPM		1000 < Speed ≤ 2000 RPM		Speed > 2000 RPM	
		Best	Average	Best	Average	Best	Average
DC	P ≤ 1 HP	0.37-1.94	0.27-3.08	0.08-0.48	0.02-0.31	0.14-0.41	0.09-0.36
Induction		0.32-3.34	0.28-2.23	0.06-0.50	0.05-0.32	0.03-0.28	0.03-0.20
Synchronous		0.02-3.43	0.07-1.40	0.05-0.66	0.01-0.30	0.02-0.25	0.02-0.23
Brushless				0.02-0.03	0.02	0.08	0.08
DC	1 < P ≤ 10 HP	0.26-0.51	0.20-0.32	0.29-0.50	0.26-0.36	0.34-1.30	0.23-0.51
Induction		0.51-0.69	0.39-0.55	0.59-1.08	0.40-0.74	0.38-0.70	0.26-0.52
DC	10 < P ≤ 100 HP	0.51-1.13	0.45-0.99	0.37-0.88	0.37-0.87	0.34-0.45	0.29-0.40
Induction		0.07-0.93	0.58-0.88	0.93-1.25	0.73-1.06	0.60-0.92	0.54-0.88

desirable features such as lighter motors, higher efficiency, acceptable manufacturing costs, and customers' technological requirements, affects the final decision.

To compare the characteristic parameters of motors from different designs, five manufacturers of induction motors are selected in this analysis with the following abbreviations as shown in the next figures.

A. Westinghouse

B. Gould

C. Electrim

D. Marathon

E. Baldor

The mean values of T/W (or P/W) in the bar chart (Fig 2-4) shows that motors from company "B" have a 35% higher ratio in RPM group A and 60% in RPM group C than motors from company "C." The trends of manufacturing motors with different powers in each company are illustrated in Figs. 2-5a and 2-5b. These graphs show the relation between T/W and power; the ratios are increased as the power becomes larger, but there are a couple of decreasing points. This does not mean that the technology of motor manufacturing becomes worse in that power range. It does mean that motor is overdesigned, e.g., equipped with a bigger frame and case than necessary. This difference becomes larger in RPM group C, which means that in high speed, more different technologies are applied.

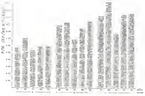
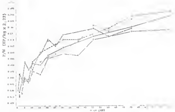
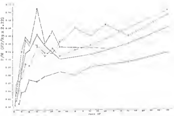


Fig. 2-4 P/W (or T/W) for induction motors from five different manufacturing companies



a. RPM group = A (1200 RPM)

Fig. 2-5 P/W (or T/W) vs power



b. RPM group = C (3600 RPM)

Fig. 2-5 - continued

Considering these factors, the difference is still large, so there are still areas to be improved in this T/W ratio which means the weight of the motor can be further reduced.

2.2.4 Characteristic Parameters for DC PM Motor

The detailed geometrical analysis was applied to the DC PM motor which has the best T/W characteristics. In Fig. 2-6, the relations among the mean values of T/W, power, and L/D ratio are plotted. In each group, the relation of T/W ratio vs. power is approximated as shown in Table 2-3, and these equations are drawn in this figure. As can be seen from this figure, the ratio group 1.3 (L/D = 1.2-1.4) has obviously maximum values of ratio. As the ratios are farther from 1.3, the values of T/W ratios decrease more although some deviations are observed.

The relation between efficiency, power, and L/D has a different maximum ratio. The efficiency has maximum relation when the ratio group is 2.0 (L/D = 1.8-2.2) as shown in Fig. 2-7 and Table 2-4.

2.3 Conclusions

From the analyses of data, the following conclusions can be obtained:

1. The DC PM motor has the highest T/W ratio among all other types of DC motors. The T/W ratio is as good as for induction motors. This suggests

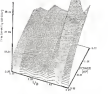


Fig. 2-6 T/W ratio vs. power in each
ratio group in DC PM motor

Table 2-3 Approximations of $T/W=f(P)$ in each group in DC PM motors

RATIO	RELATION
0.9	$T/W = 0.337P^{0.432}$
1.1	$T/W = 0.284P^{0.366}$
1.3	$T/W = 0.463P^{0.363}$
1.5	$T/W = 0.271P^{0.301}$
1.7	$T/W = 0.279P^{0.305}$
1.9	$T/W = 0.317P^{0.372}$
2.1	$T/W = 0.243P^{0.254}$
2.3	$T/W = 0.240P^{0.383}$
2.5	$T/W = 0.243P^{0.116}$

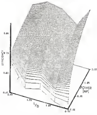


Fig. 2-7 Efficiency vs. power in each ratio group in DC PM motor

Table 2-4 Approximations of Efficiency=f(P) in each ratio group in DC PM motor

Ratio	Relations
0.85	$E_{ff} = 0.8135P^{0.1426}$
1.20	$E_{ff} = 0.7220P^{0.0948}$
1.60	$E_{ff} = 0.8055P^{0.0915}$
2.00	$E_{ff} = 0.8145P^{0.1080}$
2.40	$E_{ff} = 0.8080P^{0.0342}$

that PM motors are the most suitable for the robotic applications.

2. The best configuration (L/D) that gives its best performance is different from motors of different application.
3. The characteristic parameter in all types of motors can still be improved by adjusting the geometry or by changing the frames without introducing special techniques of manufacturing.

CHAPTER III
GENERAL APPROACH TO THE PROBLEM OF OPTIMUM DESIGN OF ELECTRIC MOTOR

3.1 Types of Optimum Design

The optimum design of an electric motor will have different results if the purpose of the motor is changed. Table 3-1 shows different types of motor applications.

The general purpose motor has continuous ratings; the output can be carried indefinitely without exceeding established limitations. The optimization of this motor is to find a motor design that has minimum initial cost or the sum of initial cost and operating cost. The former is dependent on the weight of the motor, and the latter is dependent on the losses of the motor. This design is constrained by the performance requirements specified by National Electrical Manufacturers Association (NEMA).

In special purpose motors, the motors for actuators in a robotic manipulator [43, 44] will be designed to have minimum weight because a motor with less weight will have less load inertia reflected to the next joint motors, and the motor will reduce energy consumption. The optimum design is constrained by higher peak torque in addition to the standard performance requirements. An electric vehicle [45] is similar to the

Table 3-1 Types of motor applications

		General purpose	Special purpose motors		
			Manipulator	Electric vehicle	Crane motors
Purpose		for general purpose to most of the customers	actuators for robot to move arms including other motors in next joint	traction motor for carrying the vehicle including the weight of the motor	1.load carrying 2.motion across the workshop
Power source		line or battery	line	battery	line
Duty cycle		continuous	short intermittant	long intermittent	short intermittent
Optimum design	Cost function	initial cost or initial cost + operating cost	weight	weight or weight + losses	reliability, losses
	Constraints	1.standard operating requirements 2.lower temperature rise for the same insulator class	1.standard operating requirements 2.higher peak torque 3.higher temperature rise for the same insulator class	1.standard operating requirements 2.higher peak torque 3.higher temperature rise for the same insulator class	1.safe insulation for twice rated voltage 2.bracing of coil winding & connection

robotic applications except that the power source is a battery and the operating voltage is low, which will make the objective function the sum of the weight and the losses of the motor. The crane motor [46] is another type of special purpose motor. This motor can usually be applied to the motors which carry out the task illustrated in Table 3-1. Crane motor requirements are often very severe and require a high standard of reliability to assure uninterrupted service as well as safety to personnel and equipment. Motor design must, therefore, have maximum reliability with increased cost and weight. This motor will have an ample metal section with adequate insulation and bracing of the coil windings and connections. The problem of protecting the insulator can be achieved by reducing the losses of motor. One can see how the difference in the application affects the accepted values of T/W or P/W.

3.2 General Problem of Electrical Machine Optimization

3.2.1 Modelling and Formulation of Magnetic System

The doubly excited magnetic field system with two electrical terminal pairs and rotational mechanical displacements will be modelled and optimized as a general form of electrical machine as shown in Fig. 3-1. When there is no air-gap, this system will be changed into a pot core transformer. This system will be simplified to have the following assumptions without loss of generality.

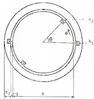

A_2
A_1
φ
θ
t_f
g
D

Fig. 3-1 Doubly excited magnetic field system

1. The reluctance of the core material is negligible.
2. The permeability of the core material is constant which means that the operating point is in the linear region of B-H curve.

Based upon above assumptions, the radial magnetic field in the air-gap can be expressed in terms of the angular variables ϕ, as in Fig. 3-1, by using the magnetic potential equation.

$$H_r 2g = N_1 i_1 \text{Sin}\phi - N_2 i_2 \text{ Sin } (\theta - \phi) \tag{3.1}$$

where

g = air-gap length

N_1 , N_2 = number of turns in primary and secondary windings

i_1, i_2 - currents in primary and secondary windings

θ - angular differences in two windings

The magnetic flux density in the air-gap is then determined from eq. (3.1) as

$$B_r = \mu_o H_r = \mu_o \left(\frac{N_1 i_1}{2g} \text{ Sin } \phi - \frac{N_2 i_2}{2g} \text{ Sin } (\theta - \phi) \right) \tag{3.2}$$

The magnetic flux linkage cut by the primary winding can be determined from the integration

$$\begin{aligned} \lambda_1 &= \int_0^{\pi} N_1 B_r LD/2 \, d\phi \\ &= L_{11} i_1 + L_{12} i_2 \text{ Cos } \theta \end{aligned} \tag{3-3}$$

where

$$L_{11} = \frac{\mu_o D L N_1^2}{2g}$$

$$L_{12} = \frac{\mu_o D L N_1 N_2}{2g}$$

In the same way the flux linkage in the secondary winding is

$$\begin{aligned} \lambda_2 &= \int_{\theta}^{\theta+\pi} N_2 B_r LD/2 \, d\phi \\ &= L_{21} i_1 \operatorname{Cos} \theta + L_{22} i_2 \end{aligned} \tag{3.4}$$

where

$$L_{21} = \frac{\mu_o N_1 N_2 LD}{2g}$$

$$L_{22} = \frac{\mu_o N_2^2 LD}{2g}$$

When the secondary winding is rotated, the mutual inductance term is changed at each angular position which generates the torque on the winding. The magnetic flux in the secondary winding is obtained from the integration

$$\begin{aligned} \phi_2 &= \int_{\theta}^{\theta+\pi} B_r LD/2 \, d\phi \\ &= \frac{\mu_o LD}{2g} (N_1 i_1 \operatorname{Cos} \theta + N_2 i_2) \end{aligned} \tag{3.5}$$

The conservation of energy in the magnetic circuit can have the following relation

$$dW' = \sum \lambda_i d i_i + \sum T_j^e d\theta_j$$

$$= \lambda_1 di_1 + \lambda_2 di_2 + T^e d\theta \qquad (3.6)$$

The coenergy stored in the air-gap is

$$W' = \frac{L_{11} i_1^2}{2} + L_{21} i_1 i_2 \text{ Cos } \theta + \frac{L_{22} i_2^2}{2} \qquad (3.7)$$

The torque which is the partial derivative of coenergy with respect to the angular displacement can be expressed as

$$T = \frac{\partial W'}{\partial \theta}$$

$$= -L_{21} i_1 i_2 \text{ Sin } \theta \qquad (3.8)$$

The average torque around the periphery inside the stator is

$$T_{av} = \frac{1}{\pi} \int_0^{\pi} T \, D\theta$$

$$= \frac{\mu_0 N_1 N_2 L D i_1 i_2}{2g} \qquad (3.9)$$

If this magnetic system can be rotated with speed w by other additional devices (ex. DC: comutator in the rotors AC: rotating magnetic field), the output power of this rotating system will be

$$P_{out} = T_{av} w$$

$$= \frac{\mu_0 A_1 A_2 J_1 J_2 D L w}{g \pi} \qquad (3.10)$$

3.2.2 Design

3.2.2.1 Input and design variables

The general approach for the optimum design of an electric motor was illustrated in Fig. 1-2. For this system design, the output power, operating voltage, and rated speed will be chosen as input parameters, and the independent design parameters must be chosen carefully. Otherwise, more constraints should be applied to this design, which will make the optimum design still more complicated. Thus, diameter (D) and length (L) of the rotor, window areas of primary and secondary windings (A_1, A_2), and current densities of both windings (J_1, J_2) can be chosen as design variables because these parameters will govern the weight, efficiency, and the cost of the motor noticeably. Other parameters, such as number of poles, number of slots per pole (N_{ss}), and air-gap length (g) are assumed to be fixed for each motor. And the thickness of the stator can be expressed as a function of design variables.

3.2.2.2 Objective function

The objective function of this system can be weight, or losses, or a combination of both, depending upon the purpose for which the machine is going to be used. The weight of this system will be

$$W = W_1 + W_2 \tag{3.11}$$

where

W_1 = iron weight

$$= \pi d_1\{(D+2g+2t_f)^2 - (D+2g)^2\}L/4 - Ld_1(2A_1+\pi D^2/4-2A_2) \quad (3.12)$$

W_2 = copper weight

$$= d_2A_1(2L+2D+4g) + d_2A_2(2L+2D) \quad (3.13)$$

The losses of this system will be

$$T_{loss} = L_1 + L_2 \quad (3.14)$$

where

L_1 = copper losses

$$= \rho J_1^2A_1(2L+2D+4g) + \rho J_2^2A_2(2L+2D) \quad (3.15)$$

L_2 = core losses

$= C_1W_1$

C_1 = core losses per unit weight

3.2.2.3 Constraints in optimisation

The optimisation of this system will be done under the following constraints.

1. Geometrical constraints
   Width of teeth in rotor and stator > 0
   Available flux path in rotor > 0
2. Magnetic constraints
   Magnetic flux density < saturation density

3. Thermal constraints

Maximum temperature < limit temperature

The geometrical constraints should be strictly adhered to; otherwise, the motor cannot be made.

3.2.3 Optimization Strategy

After all these preparations, the objective function will be optimized while maintaining the constraints. The procedure of optimization was illustrated in Fig. 1-2.

3.2.3.1 Penalty function method

There are many techniques available for the unconstrained optimization problems, but the special property of objective functions of motors made it hard to select the proper optimization algorithm. The characteristics of the motor objective function are as follows.

1. The number of turns of the winding have to be an integer value.
2. The variables of all equations are linked together, so it is very difficult to differentiate.
3. The variables are to be chosen under the constraints.
4. As a result, the objective function is not continuous.

Therefore, the algorithms using derivatives [47-49] are not suitable for this purpose. A penalty function method that transforms the constrained problem into a sequence of unconstrained minimization problems can be used in this study. There are several reasons for the appeal of penalty function formulation. One main reason, which can be observed in Fig. 3-2, is that the sequential nature of the method allows a gradual approach to the criticality of the constraints. In addition, this algorithm for the unconstrained minimization of rather arbitrary function is well studied and is generally quite reliable.

The general form of nonlinear programming with equality and inequality constraints can be stated as follows:

$$\text{Minimize } F(X) \tag{3.17}$$

$$\text{s.t.} \quad g_i(S) \le 0 \quad \text{for } i = 1,\ldots,m$$

$$h_j(X) = 0 \quad \text{for } j = 1,\ldots,n$$

Fiacco and McCormick (50) used the following expression for the augmented objective function. At the beginning of the k^{th} iteration, the augmented objective function is

$$\phi(X,r^k) = F(X) - r^k \sum_{i=1}^{m} \frac{1}{g_i(X)} + \frac{1}{\sqrt{r^k}} \sum_{j=1}^{n} h_j^2(X) \tag{3.18}$$

The penalty term, r^k, is chosen such that its value will be small at points away from the constraint boundaries and will tend to be infinity as the constraint boundaries are approached. Thus, once the unconstrained minimization of the objective function $\phi(X,r^k)$ is started from any feasible point, the subsequent points generated will always be within the feasible domain since the constraint boundaries act as a barrier during the minimization process. The penalty term in eq. (3.18) is not defined if the inequality constraint is not satisfied. This may be one problem in using eq. (3-18).

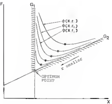

F
G_1
$\phi(x\ r)$
$\phi(x\ r_1)$
$\phi(x\ r_2)$
G_2
r smaller
OPTIMUM
POINT
X

Fig. 3-2 Illustration of penalty function method

Since this equation does not allow any constraint to be violated, it requires a feasible starting point for the search toward the optimum point. However, in many engineering problems, it may not be so difficult to find a point satisfying all the inequality constraints. If $\phi(X, r^k)$ is minimized for a decreasing sequence of values r^k, the unconstrained minima will converge to the solution of the original problem stated in eq. (3-17) as shown in Fig. 3-2.

3.2.3.2 Search algorithm

To achieve a rapid convergence to a global minimum of a N variable function, the pattern search method is used in this study. The pattern search of Hooke and Jeeves [51] is a sequential technique, each step of which consists of two kinds of moves--an exploratory move and a pattern move. The exploratory move starts X^0 as the initial point, and all coordinate directions are searched one at a time to find the descent direction. If a positive displacement of any point does not yield a reduction of the objective function, the opposite direction is tried. At the end of a series of exploratory moves, a new point X^1 can be determined. Now a pattern search is made along $(X^1 - X^0)$ direction and is continued until a minimum is found in that direction as shown in Fig. 3-3. At the end of this pattern search, the procedure is restarted until a global minimum is found.

For the minimization technique in one direction, Fibonacci search is used. Fibonacci search [52] reduces

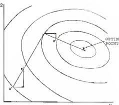

X_2
x^2
OPTIMUM POINT
x^*
x^1
x^0
X_1

Fig. 3-3 Pattern search algorithm

the interval of uncertainty about where the minimum lies by eliminating the appropriate interval after comparison of two function values in two Fibonacci points. The detailed flow chart of computer program will be shown in Appendix C. The examples of optimum design will be presented in the next sections.

3.3 Transformer Design

The brief procedure and numerical result will be discussed here, and the detailed derivations will be in Appendix A. Figure 3-4 shows the geometry of c-core transformer which is going to be optimized.

3.3.1 Preparation for Optimization [11]

The input parameters, design variables, objective function, and constraints are expressed as in Table 3-2. All these parameters are expressed as functions of design variables.

3.3.2 Optimization

The optimum design procedure of the transformer is similar to Fig. 1-2 except that the input and design parameters are different. This design will determine the main design parameters, such as the geometry (Lc, Dm, Dc), window area and current density. The detailed design can find the diameter of winding and number of turns. For the optimization of the transformer, pattern search was used

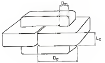


Fig. 3-4 C - core transformer

Table 3-2 Transformer design

Parameters	Descriptions
Input parameters	1.Induced output VA : S 2.Operating voltage : V
Design variables	1.Length of winding coil : L_c 2.Width of square coil : D_m 3.Width of winding part : D_c 4.Current density : J
Weight objective function	$W = W_{cp} + W_{ir}$ where W_{cp}=weight of copper winding $=2D_{cp}L_c(D_c - D_m)^2$ W_{ir}=weight of core $=D_{st}D_m^2\ (2L_c + 2D_m + 2D_c)$
Loss objective	$L = L_{cp} + L_{ir}$ where L_{cp}=copper winding losses $=\frac{\rho L_c(D_c+D_m)(D_c-D_m)^{0.75}K_uK_j^2 2^{0.25}}{(L\ D\ )^{0.25}}$ L_{ir}=core losses $=L'_{ir}W_{ir}$ (L'_{ir}=core losses per unit weight)
Constraints	1.Operating magnetic flux density 2.Maximum temperature rise of winding

3.3.3 Result

The weight function and losses function of the transformer are plotted in three dimensions as in Figs. 3-5 and 3-6. These plots show that there exists an obvious global optimum point for each objective function. The numerical results in Tables 3-3 and 3-4 show that the shape of the transformer in optimum design is not changed although the output VA is increased.

3.4 Synchronous Motor with PM Rotor

3.4.1 Preparation for Optimization [11]

The top view of a synchronous motor with a PM rotor is shown in Fig. 3-7. The input parameters, design variables, objective function, and constraints in the optimum design of a synchronous motor with a PM rotor are shown in Table 3-5, and the detailed derivations are in Appendix B.

3.4.2 Optimization

The optimum design procedure was shown in Fig. 1-2, and this design will determine the main design parameters such as diameter and length of rotor, window area of armature winding and current density of armature winding. The optimization was done using pattern search as in transformer design.

3.4.3 Result

The numerical result of optimum design for minimum weight is presented in Table 3-6. The same result with

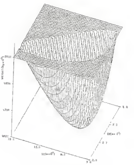


Fig. 3-5 Weight function of a transformer

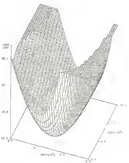


Fig. 3-6 Loss function of a transformer

Table 3-3 Numerical result of weight minimization of a transformer

Output (VA)	Weight (kg)	Losses (W)	L_c (m)	Magnetic path (m)	D_m (m)	Ratio *1	Ratio *2
100	0.89	19.58	0.057	0.220	0.0165	3.5	13.3
140	1.19	24.37	0.066	0.247	0.0180	3.7	13.7
180	1.48	28.43	0.072	0.266	0.0190	3.8	14.0
200	1.62	30.24	0.074	0.274	0.0196	3.8	14.0

Table 3-4 Numerical result of Loss minimization of a transformer

Output (VA)	Losses (W)	Weight (kg)	L_c (m)	Magnetic path (m)	D_m (m)	Ratio *1	Ratio *2
100	10.16	1.98	0.013	0.0842	0.0131	1.0	6.5
140	12.94	2.54	0.014	0.0920	0.0141	1.0	6.5
180	15.10	3.05	0.015	0.0984	0.0149	1.0	6.6
200	16.21	3.29	0.016	0.1013	0.0152	1.0	6.6

** Ratio 1 = L_c/D_m Ratio 2 = Magnetic path/D_m

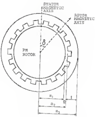

STATOR MAGNETIC AXIS
ROTOR MAGNETIC AXIS
δ
PM ROTOR
a_1
t
g
a_2
a_3

Fig. 3-7 Synchronous motor with PM rotor

Table 3-5 Design of a synchronous motor with PM rotor

Parameters	Description
Input parameters	1.Output power P_{out} 2 Operating voltage = V 3 Rated speed w 4 Number of poles N_p
Design variables	1.Diameter of rotor . D 2 Length of rotor L 3.Armature window area . A_a 4 Current density J
Weight objective	$W = W_{ro} + W_{ac} + W_{aw}$ where W_{ro}=rotor weight $= \pi D_{ro} D^2 L/4$ W_{ac}=armature core weight $= D_{ac} L [\{\pi(D+2g+2t)^2 - \pi(D+2g)^2\}/4$ $- 6mn_{as} N_{ap} A_a]$ W_{aw}=armature winding weight $= 6mD_{cp} n_{as} N_{ap} A_a L k_a$ $+ 3D_{cp} n_{as} N_{ap} A_a k_s \pi D$
Constraints	1.Width of armature teeth : $W_{at} > 0$ 2.Magnetic flux density $B_{at}, B_{ay} < B_{max}$ 3 Window area of armature winding $A_a > 0$ 4.Full load temperature rise $\leq T_{limit}$

Table 3-6 Numerical result of weight minimization of synchronous motor with PM rotor

Power (HP)	Pole pair	Weight (kg)	Length (m)	Magnetic path length	Diameter (m)	Ratio 1*	Ratio 2*
10	1	46.8	0.14	0.859	0.226	0.62	3.8
	2	81.9	0.18	0.719	0.257	0.68	2.8
	3	113.0	0.193	0.676	0.282	0.69	2.4
20	1	85.1	0.183	1.041	0.274	0.67	3.8
	2	149.0	0.213	0.874	0.312	0.68	2.8
	3	205.2	0.239	0.823	0.343	0.69	2.4

** Ratio 1 = Length/Diameter

Ratio 2 = Magnetic path length/Diameter

transformer was found that the shape of motor is not changed; in other words, the ratio between diameter and length of rotor is the same for a series of different power ranges.

3.5 Conclusions

From all these analyses and results of optimization, we can get the following conclusions:

1. All machines have their own optimum configurations.
2. The optimum configuration is not changed although the power rating is changed.

CHAPTER IV
STRUCTURE OF DESIGN PROVIDING MINIMUM WEIGHT/LOSSES OF ENERGY

The general problem of electrical machine optimization was shown, and the simplified models in transformer and synchronous motor were optimized in Chapter III. This chapter will optimize the DC PM motor with respect to weight, losses of energy and combination of both objective functions in the normal structure of a motor. The motor with an "inside-out" structure will be optimized to compare the advantages of this structure.

4.1 The Specific Characteristics of DC PM Motors

Conventionally wound field DC motors have been the main DC energy conversion devices since the discovery of electromechanical motion. Recently, the permanent magnet (PM) motors challenged the traditional wound field DC motors in all sizes up to 5 HP [53]. For the last thirty years, the use of PM fields in small DC motors has been increasing to the point where a majority of the motors are now changed into PM motors. Furthermore, the development of the new PM materials also evolved and broadened their applications.

Prior to the PM motors, the three popular types of DC motors for industrial applications were shunt, series, and compound motors. Designers often made compromises in their equipment in order to use one of these three motors. For example, gears and pulleys were often added to reduce the speed of a series motor enough to make the portable equipment practical.

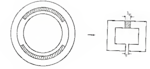
For the proper selection or design, it is necessary to understand the characteristics, advantages, and shortcomings of each type of motor. Table 4-1 shows the collected information of the characteristics of wound field motors. As can be seen from this table, the wound field DC motors have a number of shortcomings.

To see the superior properties of PM materials, let us make a comparison between the PM and wound field methods as illustrated in Fig. 4-1 and Table 4-2.

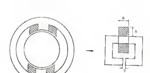
The volume of field part is proportional to the dimension, D, in wound field method, and is proportional to the square of core dimension, D^2, in PM field method. The coefficient of volume equation in the PM field is much smaller than that in the wound field method. Thus, the volume of the PM motor is smaller than that of the wound field motor in small power applications. If the power is very low, the volume of the PM motor is much smaller than that of wound field motor. This coincides with the fact that most of the PM motor applications have been confined

Table 4-1 Comparison of wound field DC motors

Kinds	Advantages	Disadvantages
Shunt motor	1. Relatively good speed regulation up to 250 % 2. Variety of control (voltage & field cont.) 3. Capability of high power application	1 Small starting torque (armature reaction)
Series motor	1. Highest starting torque 2. Higher power per weight ratio 3. Wide speed capability	1. Bad speed regulation 2 Over-speed at light load 3 Small power use
Compound motor	1. Better than pure shunt motor	1. Complicated control circuit needed



a. PM excitation



b. Wound field excitation

Fig. 4-1 Comparison between PM field and wound field methods

Table 4-2 Comparison of PM and wound field methods

		Wound field	PM field	
Basic equation		$H_g g = N_a I_a$	$H_g g + H_m l_m = 0$ $B_m = B_r + \quad H_m$ $B_m A_m = q B_g A_g$	
Volume of field part		$V_c = 4A^2 D + 4A^3$	$V_m = D^2 l_m$	
B_g (G)	1000	$A = 0.446$ $V_c = 0.8D + 0.35$	Ferrite	$V_m = 0.05D^2$
			$SmCo_5$	$V_m = 0.01D^2$
	2000	$A = 0.631$ $V_c = 1.59D + 1.00$	Ferrite	$V_m = 0.18D^2$
			$SmCo_5$	$V_m = 0.03D^2$
	4000	$A = 0.892$ $V_c = 3.18D + 2.84$	Ferrite	X
			$SmCo_5$	$V_m = 0.09D^2$
	8000	$A = 1.260$ $V_c = 6.37D + 8.00$	Ferrite	X
			$SmCo_5$	$V_m = 1.60D^2$

to smaller power ranges; the motors for robotic applications are within these power ranges.

We can also see from this table that as the magnetic flux density increases, the volume of the wound field motor is increased quite linearly, but in PM field motors, the volume is increased much faster. So the operating point of the PM motor is normally very close to the maximum energy product point.

The advantages of the DC PM motor over the wound field motor are listed below [53-55] with the result of comparison from the database as shown in Table 4-3:

1. Size: The outer diameter of a PM motor is reduced from 10% to 30%. And in our database, the diameter of a DC PM motor in 1 HP is 30% less than that of a DC shunt motor and 16% less in 2 HP. The reasons for the smaller outside diameter are that there is no bulky field, and the magnets can be assembled by cementing the segments to the shell.
2. Weight: A weight reduction up to 30% can be expected in the motor due to the smaller diameter with the same size of armature. And in our database, the weight of the PM motor is 46% less than that of a shunt motor in 1 HP and 17% less in 2 HP
3. Efficiency: An increase in the efficiency by up to 15% occurs because the field is generated by a permanent magnet and does not have to be energized

Table 4-3 Comparison of DC PM motor and shunt motor from the database

	1 HP		2 HP	
	PM	Shunt	PM	Shunt
Diameter (m)	0.149	0.187	0.179	0.213
Weight (kg)	23.2	53.2	64.8	78.2
Efficiency(%)	80.5	79.7	80.2	75.3

4. Temperature: Higher efficiency means cooler operation for the same power, so the PM motor concept is suitable for the equipment where heat losses are problems in operation.
5. Armature reaction: The PM pole piece has a permeability very close to that of air. And when the armature reaction flux leaves the armature in a PM motor, the flux lines are resisted by a low permeability material. In other words, the PM motor has a very large effective air-gap. So the armature reaction does not distort the main field at the overload condition in a PM motor as it does in a wound field motor. As a result, it offers a relatively high starting torque (650% of rated torque), and the performance of a PM motor is a straight line over the operating range.

Considering all these above, the conversion to the PM motor is quite natural.

4.2 Comparison of Permanent Magnet Materials [56-59]

The permanent magnet materials have been developed to have higher remanent flux density, higher coercive force, and higher energy product. There are three major PM families in use--alnico, ferrites, and rare earth magnet. We will consider the characteristics of each PM material.

Demagnetization characteristics of those PM materials most commonly used in rotating electrical machines are shown in Fig. 4-2 together with some new materials of interest. Table 4-4 illustrates the comparison of PM materials.

Alnico magnets have been traditionally employed in electrical machines. The high remanence type, Alnico 5-7, offers potentially higher flux density with good temperature characteristics. However, the very low coercive force requires a very long magnet. Furthermore, the knee in the curve occurs only slightly below the remanent flux density. This will result in the operating point's being forced down the steep part of the demagnetization curve with only a small additional demagnetizing field due to an armature reaction. Another problem concerning this low coercive force is that this magnet must be remagnetized after assembling it to the machine.

Ceramic ferrites are most commonly used in the production of PM motors due to their cost advantage, availability and ease of fabrication into the stator. The demagnetization curve is quite linear over most of the quadrant, and the very low permeability ensures that recoil property will be back along the same curve rather than within the curve as with Alnico's. Ferrites may be magnetized before assembling because of high coercive force. But the remanent flux density is very low, and they are frequently designed to operate well above the maximum energy point, to get more flux at the expense of additional magnet length.

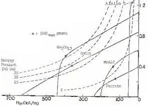


Fig. 4-2 Demagnetization characteristics of PM materials

Table 4-4 Comparison of PM materials

KINDS	ADVANTAGES	DISADVANTAGES
ALNICO5-7	1.HIGHEST B_r (1.22 T) 2.GOOD TEMPERATURE CHARACTERISTICS	1.LOWEST H_c (70 kA/M) 2 LAGER NEEDS LONGER MAGNET 3.REMAGNETIZATION AFTER ASSEMBLY
FERRITE	1.CHEAPEST 2.EASY FABRICATION 3.HIGHER H_c (240 kA/M) 4.LOWER $\mu r \approx 1$	1.LOWEST B_r (0.4 T) REQUIRES LONGER MAGNET
RARE EARTH MAGNET	1 HIGHEST ENERGY PRODUCT (144 kJ/M^3) 2.HIGHEST H_c (630 kA/M) 3.HIGHER B_r (0.85 T) 4.SUITABLE FOR "INSIDE-OUT" MOTOR	1.HIGHEST MATERIAL COST

Rare earth PM materials [60-61], were developed about 20 years ago, and their properties, which are almost the best in all areas, are still improving. The properties of these materials which intrigue the motor designer most are the following.

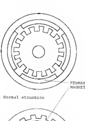
1. Highest energy product: 144 kJ/m
2. Highest coercive force: 388 kA/m
3. Highest remanent flux density: 0.85 T
4. High curie temperature: 775°C

The only disadvantage is the price of materials, which limits the design criterion of this type to operation at the maximum energy point to minimize the magnet volume. With this rare earth material, it is now possible to place the PM field in the rotor.

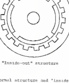
4.5 Design

Figure 4-3 shows the normal structure and "inside-out" structure of the DC PM motor. The "inside-out" structure is the inverted structure where the armature part is in the stationary part and PM is in the rotor as shown in the figure. The difference in detailed design will be discussed later.

Based upon the general design structure in Fig. 1-2, the optimum design procedure is organized as shown in Fig. 4-4 which illustrates the optimization algorithm, SUMT and pattern search, discussed in Chapter III. Most of the


PERMANENT
MAGNET

a. Normal structure



b. "Inside-out" structure

Fig. 4-3 Normal structure and "inside-out" structure of DC PM motor

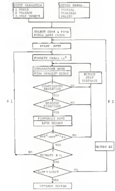

INPUT PARAMETER
1 POWER
2 VOLTAGE
3 POLE NUMBER
DESIGN PARAM
INITIAL
FEASIBLE
VALUES
SELECT CORE & FIND
FIELD LOSS CURVE
START SEPT
PENALTY PARA. r^k
EXPLORATORY MOVE
FIND DESCENT DIREC
REDUCE
STEP
DISTANCE
CONSTRAINTS
SATISFIED
NO
YES
DESCENT
DIRECTION
NO
YES
# 1
# 2
FIBONACCI MOVE
LINE SEARCH
$OPT^k - OPT^{k+1} \le \epsilon$
NO
YES
ADJUST XJ
NO
PENALTY $\le \epsilon_1$
YES
TEMP = LIMIT
NO
YES
OPTIMUM DESIGN

Fig. 4-4 Basic structure of optimum motor design

procedures are the same as the general structure shown in Fig. 1-2. This figure explains the first step design which decides the optimum motor configuration.

After the input parameters of DC PM motor design are given, which are the specifications of motor performance, the magnetization and core loss curves for selected core are determined using the Chebyshev polynomial approximation [62]. When the SUMT is started under given penalty parameter r^k, the pattern search which consists of an exploratory move and a Fibonacci move was used to find the minimum weight or loss objective function. After the series of minimization procedures, the convergence criterion is checked. If the criterion is satisfied, the optimum design parameters can be obtained, otherwise, this optimization procedure (#1 in Fig. 4-4) will be reiterated until the criterion is satisfied.

With these obtained design parameters, the temperature rise of armature winding is to be checked because the armature winding is the source of heat and has the highest temperature in the motor. If the temperature rise is not the same as the maximum limit, the current density is adjusted and the optimization procedure (#2 in Fig. 4-4) will be started again until it reaches the temperature limit.

Now we will consider the actual design of the DC PM motor with normal and "inside-out" structure. Although both structures are different in configurations, their basic magnetic circuits are the same as shown in Fig. 4-5.

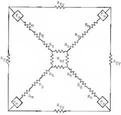

R_{fy}
F_m
F_m
R_m
R_m
R_g
R_g
R_t
R_t
R_{fy}
R_{ay}
R_{fy}
R_t
R_t
R_g
R_g
R_m
R_m
F_m
F_m
R_{fy}

Fig 4-5 Equivalent magnetic circuit of four pole DC PM motor

4.3.2 Normal Structure of Motor

4.3.1.1 Selection of design parameters

The flow chart of computing the objective function which illustrates the design procedure is shown in Fig 4-6. The input parameters are power (P_{out}), voltage (V), rated speed (w), and number of Poles (N_p) for general purpose motors. The motor for the robotic actuator will have rated torque (T_r) and number of poles (N_p) as input parameters.

It is very important to select the design variables properly; otherwise, the optimization problem becomes complicated and sometimes hard to converge to optimum value. The design parameters finally selected in DC PM motors are shown below

1. Rotor diameter: D
2. Rotor length: L
3. Armature window area: A_a
4. Width of magnet: X_{lm}
5. Thickness of field yoke: t_f
6. Current density: J_a

4.3.1.2 Induced voltage computation

From the given voltage and power, the diameter of winding can be determined using the following basic equations in steady state.

$$V = E_g + I_a R_a \tag{4-1}$$

$$V I_a = P_{out} + I_a^2 R_a \tag{4.2}$$

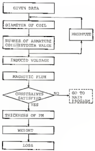

GIVEN DATA
DIAMETER OF COIL
RECOMPUTE
NUMBER OF ARMATURE COIL INTEGER VALUE
INDUCED VOLTAGE
MAGNETIC FLUX
CONSTRAINTS SATISFIED
NO
GO TO MAIN PROGRAM
YES
THICKNESS OF PM
WEIGHT
LOSS

Fig.4-6 Flow chart of designing procedure of DC PM motor

where

$$I_a = J_a \pi D_w^2 a/4 \tag{4.3}$$

D_w = diameter of armature winding

J_a = current density in armature winding

E_g = induced voltage

a = number of parallel paths

$P_{out} = E_g I_a$

The resistance of armature winding, R, can be expressed as

$$R = \rho L/S$$

$$= \frac{\rho M_{1t} A_a N_{as} m}{(a\pi D_w^2/4)^2} \tag{4.4}$$

where

ρ = resistivity of winding material

M_{1t} = mean turn length of armature winding

A_a = armature window area

N_{as} = number of armature slots per pole

m = number of pole pairs

Substituting eq. (4.4) into eq (4.2) yields

$$VJ_a \pi D_w^2 a = P_{out} + L_{cp} \tag{4.5}$$

where

$L_{cp} = J_a^2 \rho M_{1t} A_a N_{as} m$, copper losses

From eq (4 5), the diameter of windin [illegible]

$$D_w = \sqrt{\frac{(P_{out} + L_{op})}{VJ_a a\pi/4}} \qquad (4\ 6)$$

After the diameter of armature winding is obtained, the induced voltage and magnetic flux can be computed respectively

$$E_g = V - I_a R_a \qquad (4\ 7)$$

$$\phi = \frac{\pi a E_g}{N H_p w} \qquad (4\ 8)$$

where

N = total number of turns

$= m h_{es}$ Integer $(A_s/(\pi D_w^2/4))$ (4 9)

w – angular speed

4 3 1.3 Thickness of PM

The equivalent magnetic circuit of PM motor was shown in Fig. 4-5 The reluctance of air-gap, magnet, teeth, armature yoke, and field yoke can be expressed as a function of geometrical parameters respectively

$$R_g = \frac{g_e}{\mu_0 A_g} \qquad (4\ 10)$$

$$R_m = \frac{T_m}{\mu_r \mu_0 A_m} \qquad (4\ 11)$$

$$R_t = \frac{D_{ss}}{\mu_1 \mu_0 A_t} \qquad (4\ 12)$$

$$R_{ay} = \frac{P_{ay}}{\mu_2 \mu_0 A_{ay}} \tag{4.13}$$

$$R_{fy} = \frac{P_{fy}}{\mu_3 \mu_0 A_{fy}} = \frac{(D + 2g_e + t_f)/N_p}{\mu_3 \mu_0 A_{fy}} + \frac{t_m}{\mu_3 \mu_0 A_{fy}} \tag{4.14}$$

g_e = effective air-gap (63)

$$= g \frac{D_{as} + W_{as}}{D_{as} + W_{as}/(1 + 5g)}$$

g = air-gap lentgh

A_g, A_m, A_{ay}, A_t, A_{fy} = cross sectional area of air-gap, magnet, armature yoke, teeth, field yoke

T_m = thickness of magnet

$\mu_1 \mu_2 \mu_3$ = permeability of teeth, armature yoke, and field yoke

And the magnetic circuit equation can be expressed as

$$F_m = \phi(2R_m + 2R_g + 2R_t) + (R_{ay} + R_{fy})\phi/2 \tag{4.15}$$

where

$$F_m = 2R_m \phi_{r1} \tag{4.16}$$

$$\phi_{r1} = B_r X_{1m} L$$

From eq. (4.10) - eq. (4.16), the thickness of permanent magnet that can generate the magnetic flux will be determined.

$$T_m = \frac{F_1 + F_3}{\frac{2B_r}{\mu_r \mu_0} - F_2} \tag{4.17}$$

where

$$F_1 = \phi(2R_g + 2R_t = R_{ay}/2)$$

$$F_2 = \phi(\pi/(2\mu_3\mu_0 A_{fy}) + 2/(\mu_r\mu_0 A_m)$$

$$F_3 - \frac{\pi(D + 2g_e + t_f)}{4\mu_3\mu_0 A_{fy}}$$

4.3.1.4 Approximations of magnetization and loss curves

To determine exactly the core losses and the magnetization curve, the Chebyshev polynomial and curve fitting are used The results of approximations are shown in Figs 4-7 and 4-8. The actual data represented by '.' are quite close to the approximated data.

4.3.1.5 Objective function

The objective function of the design in this study will use the weight, losses, or combination of both

1. Weight

Armature part

$$W_a + W_{aw} + W_{ai} + W_{ay} + W_{at} \tag{4-18}$$

where

W_{aw} = armature winding weight - $D_{cp}M_{lt}NA_a$

W_{ai} = insulator weight

$= D_{in}L\pi(D_{ae}W_{as} - A_s)N_pN_{ee}$

W_{ay} = yoke weight = $D_{ir}L\pi(D - 2D_{ae})^2/4$

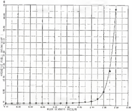


Fig. 4-7 Approximation of magnetization curve

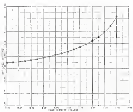


Fig. 4-8 Approximation of loss curve

W_{at} = teeth weight

$= D_{ir}D^2L\pi/4 - W_{ay} - N_pN_{as}D_{ir}D_{as}W_{as}L$

Field part

$$W_f = W_{fp} + W_{fy} \tag{4.19}$$

where

W_{fp} = PM weight = $N_pD_{pm}V_{pm}$

W_{fy} = yoke weight

$= \frac{D_{ir}L\pi}{4}\{(D+2g+2T_m+2t_f)^2 - (D+2g+2T_m)^2\}$

Total weight

$$W_t = W_a + W_f \tag{4.20}$$

2 Losses

Copper losses of armature winding

$$L_{cp} = \rho J_a^2M_{lt}A_aN_{as}m \tag{4.21}$$

Core losses

$$L_{cr} = L_{at} + L_{ay} \tag{4.22}$$

where

L_{at} = teeth core losses = $L_{s1}W_{at}$

L_{ay} = yoke core losses = $L_{s2}W_{ay}$

**L_{s1}, L_{s2} = teeth, yoke core losses per unit weight

Total losses

$$T_{loss} = L_{cp} + L_{cr} \tag{4.23}$$

3. The combination of these two cost functions can be expressed using weighing factor λ.

$$OBJ = (1 - \lambda)W_t + \lambda T_{loss} \tag{4.24}$$

4.3.1.6 Constraints

The constraints for the optimum design of a DC PM motor which are similar to those in Chapter III, are listed below.

1. The width of armature teeth and armature window area are positive.

   $W_{st}, A_e > 0$

2. The magnetic flux density in teeth, armature yoke, and field yoke is less than the limit.

   $B_t, B_{et}, B_{fy} \le B_s$

3. The temperature rise of armature winding is the same as the temperature limit

Among those constraints, the temperature rise is not computed easily because the heat dissipation is related to its structure, environmental conditions, and time. This will be discussed in detail in the next section.

4.3.1.7 Thermal computation

1. Heat transfer between rotor and stator. The heat transfer characteristics of air-gap in electrical motors are

the least studied of the many heat flow paths in an electrical rotating machine. Martinelli [64] has derived an expression for the heat transfer to a fluid flowing between two parallel plates with a constant heat rate at each plate. But that expression is valid only when the fluid is being symmetrically heated from each plate. In our case, the fluid is heated from one side and cooled from the other.

Seban [65] and Garley [66] modified that expression of the Nusselt number for heat transfer across air-gap and compared this with other experimental data. His modified formula will be used in this study.

Now consider the concentric cylinder flow in which no axial flow occurs as shown in Fig. 4-8, which illustrates the basic structure of the motor for this thermal analysis. In this case, the flow is actuated only by the rotation of one of the cylinders and can be characterized by a Reynolds number and by a dimensionless curvature factor. The Reynolds number is defined in the usual way.

$$R_{eg} = \frac{\text{(equivalent diameter)(mean velocity)}}{\text{kinetic viscosity}}$$

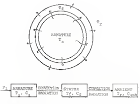
$$= \frac{g u_p}{\nu} \tag{4.25}$$

where

g = air-gap length

u_p = peripheral velocity and rotor

The dimensionless Nusselt number for heat transfer across air-gap was derived to be [65]


T_f
T_r
ARMATURE
T_a
F_l
ARMATURE
T_a, C_a
CONVECTION
RADIATION
STATOR
T_f, C_f
CONVECTION
RADIATION
AMBIENT
T_r, C_{amb}

F_l = Total losses
T_a = Armature temperature
T_f = Field part temperature
T_r = Ambient temperature
C_f = Field part heat capacity
C_a = Armature heat capacity
C_{amb} = Ambient air heat capccity

Fig. 4-9 Heat flow of DC PM motor in normal structure

$$N_{ug} = \frac{f_r P_r R_e (c_f/2)^{0.5}}{10(P_r + \ln(1+5P_r) + 0.5\ln R_{eg}(c_f/2)^{0.5}/120]} \quad (4.26)$$

where

f = friction factor

(f = 0.715 when $R_{eg} \leq 10,000$)

P_r = Prandtl number

R_{eg} = Reynolds number

c_f = skin friction coefficient

The relation between skin friction coefficient and the Reynolds number approximated from the measured data is to be

$$c_f = (10/R_{eg})^{0.667} \quad (4.27)$$

The heat transfer coefficient by convection across an air-gap is

$$h_g = \frac{kN_g}{2g} \quad (4.28)$$

The power transfer by convection across air-gap is

$$P_{a1} = h_g A_r (T_a - T_f) \quad (4.29)$$

The heat transfer across the air-gap by radiation is

$$P_{a2} = 5.71 \times 10^{-8}((T_a + 273)^4 - (T_f + 273)^4)A_r \varepsilon \quad (4.30)$$

The total heat transfer across the air-gap is the sum of these two heats.

$$P_a = P_{a1} + P_{a2}$$

2. Heat transfer from motor to ambient air [67]. For convection in still air, the heat transfer coefficient can be used from [68], converted to the units used in this study

$$h = 14.646\frac{(T_f - T_r)^{0.25}}{D_1^{0.26}T_{f1}^{0.37}} + 9.072\times10^{-4}T_{f1}0.743/D_1 \qquad (4.31)$$

where

D_1 = equivalent diameter of model

T_{f1} = air film temperature

$$= \frac{T_f + T_r}{2} + 273$$

So the heat dissipation from the motor and the air by convection is

$$P_{o1} = hA_r(T_f - T_r) \qquad (4.32)$$

And the heat dissipation by radiation is

$$P_{c2} = 5.71 \times 10^{-8}\varepsilon[(T_f + 273)^4 - (T_r + 273)^4]A_r \qquad (4.33)$$

The total heat transfer from the surface of the motor to the environmental air is then

$$P_c = P_{c1} + P_{c2} \tag{4.34}$$

3. Temperature rise computation. When all these heat dissipations are computed, the temperature rise of armature winding, T_a, and field part, T_f, can be determined by solving the thermal differential equations.

$$\frac{dT_a}{dt} = (P_1 - P_a)/C_a \tag{4.35}$$

$$\frac{dT_f}{dt} = (P_a - P_c)/C_f \tag{4.36}$$

These equations are the nonlinear differential equations; so the numerical method, Runge-Kutta predictor corrector method [63] can solve these equations. The motors for robotic applications have a short intermittent duty cycle (Chapter III); so the temperature rise is computed on a one-hour time period, which is different from the continuous rating in general purpose motors shown in Table 3-1.

4.3.2 Inside-out Structure

Since the basic structures of normal and "inside-out" are quite similar, this section will discuss just the differences between these two structures.

The thickness of the PM magnet in this structure of motor can be obtained using the same magnet circuit equation except for the difference of reluctances by the change of structure. The changed reluctances are armature and field yoke reluctances:

$$R_{ay} = \frac{P_{ay}}{\mu_2 \mu_0 A_{ay}} \tag{4.37}$$

$$R_{fy} = \frac{P_{fy}}{\mu_3 \mu_0 A_{fy}} \tag{4.38}$$

where

$P_{ay} = \pi(D + 2(g_e + D_{as}) + T_a)/N_p$

$A_{ay} = T_a L$

$P_{fy} = \frac{\pi(D + D_{ab})}{2N_p}$

T_a = thickness of armature yoke

Using eq. (4.37) and eq. (4.38), the thickness of PM magnet can be expressed by the following equation

$$T_m = \frac{1.3\phi(2(R_g + R_t) + 0.5(R_{ay} + R_{fy}))\mu_r \mu_0 A_m}{2B_r X_{lm} L - 2\phi} \tag{4.39}$$

The weight objective function of this motor is

$$W_t = W_a + W_f \tag{4.40}$$

where

$W_a = W_{aw} + W_{ai} + W_{ay} + W_{at}$

$W_{aw} = D_{cp} M_{lt} N A_a$

$W_{ay} = \frac{D_{ir} L \pi}{4} \{(D+2g+2D_{as}+T_a)^2 - (D+2g+2D_{as})^2\}$

$W_{at} = \frac{D_{ir} L \pi}{4} \{(D+2g+2D_{as})^2 - (D+2g)^2\} - N_p N_{as} D_{ir} D_{as} W_{as} L$

$W_{ai} = D_{in} L (D_{as} W_{as} - A_a) N_p N_{as}$

The losses of this motor are

$$T_{loss} = L_{cp} + L_{cr} \tag{4.41}$$

where

$$L_{cp} = \rho J_a^2 M_{1t} A_a N_{as} m$$

$$L_{cr} = L_{at} + L_{ay}$$

$$L_{at} = L_{st} W_{at}$$

$$L_{at} - L_{ay} W_{ay}$$

$(L_{st}, L_{ay}$ = core losses in teeth, yoke)

The heat dissipation of this structure is different from that of a normal structure motor. The heat generation in the armature is dissipated to the field part and air with a larger surface area so the effective heat dissipation area is greatly increased (Fig. 4-10). As a result, the thermal equation can be changed as follows:

$$\frac{dT_a}{dt} = (P_1 - P_a - P_c)/C_a \tag{4.42}$$

$$\frac{dT_f}{dt} = P_a/C_f$$

4.4 Optimization

This study is concentrated on weight minimization ($\lambda = 0$) because of the importance of weight in manipulator operation mentioned in Chapter I. Based on the design procedure illustrated in Fig. 4-4, the computer program is

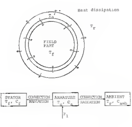

Heat dissipation
T_a
T_r
FIELD PART
T_f
STATOR
T_f, C_f
CONVECTION
RADIATION
ARMATURE
T_a, C_a
CONVECTION
RADIATION
AMBIENT
T_r, C_{amb}
P_1

Fig. 4-10 Heat dissipation of DC PM motor in "inside-out" structure

organized and the weight objective function is optimized using SUMT and pattern search method. This design procedure will be explained using the structure of a computer program.

4.4.1 Computer Program

4.4.1.1 Main program

The main computer program will do the following tasks:

1. Determine the purpose of DC PM motor if the motor is for general purpose or for multilink manipulator.
2. Read in the input data and initial feasible values of design variables.
3. Find the magnetization and core loss curves of selected core material.
4. Control the SUMT by the penalty factor
5. Check the temperature rise, adjust the current density, and repeat above procedures 3 and 4 until the temperature rise is the same as the limit.
6. Print the specifications of the designed motor.

4.4.1.2 Subprogram "PATTEN"

The subprogram "PATTEN" will find the minimum point by the following procedures:

1. The subprogram 'EXPLOR' will find the descent direction by trying all the directions of design variables that can reduce the objective function. If the search could not reduce the objective

function, the new values of design are replaced by the former values before the search.

2. If the exploratory move fails, the search step distances will be reduced and the exploratory move tried until the descent direction is found.
3. The pattern move will be done by the subprogram "FIBONA." This subroutine will find the minimum point using a Fibonacci search which is an effective line search method.
4. The convergence criterion can be satisfied if the three successive results of pattern search are within a satisfactory limit, otherwise the above procedure will be reiterated until the criterion is satisfied.

4.4.1.3 Subprogram "DESIGN"

The objective function can be determined from this subprogram "DESIGN" which will find the performance parameters and check the constraints. The computing procedures are shown in Fig. 4-6. The detailed flow charts of optimization and computer program are in Appendices C and D.

4.5 Result of Optimization

4.5.1 Convergence Check

The penalty function method and pattern search method used in this computer program of optimum design show rapid convergence to the optimum values. Figure 4-11 shows the

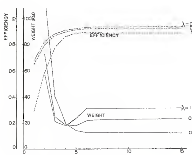


Fig. 4-11 Convergence mode of optimization

convergence speed of optimum design of a 5 N-M motor. After six iterations, all of the design parameters are very close to optimum values, but the optimization procedures is continued for up to 15 iterations to increase the accuracy of the results. Optimization with different starting points was tried, and the results are within 0.2% difference.

4.5.2 Computer Output

The specifications of the designed motor are printed with the format shown below:

```
Result of optimization

1.  Input Data

        Torque (N-m) ....................      5.000
        Number of poles .................      2
        Maximum speed (RPM) .............   1600
        Power (HP) ......................      1.26

2.  Design Parameters

        Rotor diameter (cm) .............      8.529
        Rotor length (cm) ...............      7.373
        Slot area (cm^2) ................      0.767
        Magnet width (cm) ...............      6.964
        Stator thickness (cm) ...........      1.134
        Current density (A/cm^2) ........    367.800
        Ratio (L/D) .....................      0.664

3.  Electrical Specifications

        Number of turns per slot .......     60
        Number of slots per pole .......     10
        Total number of turns ..........    600
```

Diameter of armature coil (cm)	0.176
Resistance of armature coil (Ω)	1.760
Inductance of armature coil (mH)	28.470
Induced voltage (volt)	103.514
4. Magnetic Specifications	
Flux (WB)	0.00784
Flux density (TESLA)	
Magnet	0.430
Air-gap	0.430
Armature teeth	1.800
Armature yoke	1.771
Field yoke	1.698
5. Loss (watt)	
Copper loss	105.885
Core loss	32.516
Mechanical loss	13.840
Total loss	152.242
Efficiency (%)	86.093
6. Weight (kg)	
Armature (ROTOR)	5.419
Winding	3.409
Yoke	1.031
Teeth	0.906
Field (STATOR)	2.367
Magnet	0.267
Yoke	2.096

Total weight	7.781
7. Acceleration (rad/sec^2)	507.347
Torque (N-m)	5.000
Inertia (kg-m^2)	0.00986
8. Performance Constant	
Voltage constant (K_e·V·sec/rad)	0.549
Torque constant (K_T N-m/A)	0.548
Electrical time constant (msec)	22.596
Mechanical time constant (msec)	41.225

4.5.3 Numerical Results with Different Torques

Table 4-5 shows the numerical results of optimum design of weight minimization ($\lambda = 0$) in a series of different torques. In lower torque ranges, the motor is very small, and it is hard to satisfy the following constraints with the small rotor diameter:

1. The width of slot in the armature is positive to put the armature winding in it.
2. The tooth width in the armature is positive and is larger than some value to make the magnetic flux density in the tooth less than saturation density.

As a result, the diameter of a rotor is to be increased to satisfy those constraints. As the torque becomes larger, those constraints can be satisfied easily with this diameter. When the torque of the motor is larger than 5 N-m (1.26 HP with 1800 RPM), the ratio remains constant.

Table 4-5 Numerical result of optimization in different torques

TORQUE (IN M) / POWER (HP)	RATIO(L/D)	WEIGHT(Kg)	LOSSES(W)	EFF.(%)	INERTIA $(Kg-M^2)$	ACCELERATION $(RAD./SEC^2)$
0.1/0.03	0.228	0.59	21.50	46.7	0.00031	323.5
0.3/0.08	0.342	1.22	33.93	62.5	0.00080	373.4
0.5/0.13	0.403	1.70	43.28	68.5	0.00126	396.1
0.7/0.18	0.446	2.12	51.22	72.0	0.00171	409.2
1.0/0.25	0.503	2.68	61.63	75.4	0.00235	425.2
3.0/0.76	0.722	5.55	113.51	83.3	0.00635	472.8
5.0/1.26	0.864	7.78	152.24	86.1	0.00986	507.3
7.0/1.77	0.882	9.78	186.09	87.6	0.01403	498.9
10 /25 3	0.891	12.62	234.20	88.9	0.02127	470.2
15 /3.79	0.891	17.03	308.14	90.2	0.03508	417.6
20 /5.05	0.888	21.11	375.32	90.9	0.05037	397.1
40 /10 11	0.885	35.48	608.59	92.5	0.12049	332.0

4.5.4 Comparison with Different Material and Structure

The DC PM motor is designed in different magnet materials, ferrite and $SmCo_5$, and in different structures, normal and "inside-out." The results of these designs are illustrated in Table 4-6. From this table, one can see the following differences.

1. In normal structure, the $SmCo_5$ motor shows 30% less weight, 3% higher efficiency, and 35% higher no load acceleration than does the ferrite motor.
2. The $SmCo_5$ PM motor with "inside-out" structure has 200% increase of no load acceleration over the normal structure of a $SmCo_5$ motor. As a result, more motor torque can be delivered to the load.

4.6 Conclusions

Based upon the analysis and results of DC PM motor optimum design, we can get the following conclusions:

1. The DC PM motor has better characteristics than the wound motor, as shown in Table 4-6.
2. The DC PM motor has a unique shape, but the constraints modify the shape to make a feasible design.
3. The most desirable material for an actuator in robotic application is a rare earth PM material.
4. The suggested structure of a motor for robotic application is the "inside-out" structure with a rare earth PM magnet.

Table 4-6 Comparison of optimization result with different material and different [illegible]

MATERIAL		D(CM)	L(CM)	L/D	W(Kg)	EFF(%)	Acc.
	2	8.37	10.77	1.286	9.49	84.2	413
FERRITE	4	9.88	6.70	0.678	7.39	88.6	349
	6	11.69	4.96	0.424	7.13	89.7	245
$SmCo_5$ (NORMAL)	2	8.53	7.37	0.860	7.78	86.1	507
	4	10.00	4.68	0.470	5.77	91.5	465
	6	12.23	3.33	0.260	5.46	92.5	307
$SmCo_5$ (inside-out)	2	6.19	7.08	1.130	7.19	86.3	3185
	4	8.90	3.96	0.445	5.29	90.8	1311
	6	11.15	3.00	0.271	5.08	92.0	703

MOTOR : TORQUE 5 N-M

SPEED 1800 RPM

** note

The DC shunt motor with the same power rating has the following values in each parameter.

L/D = 2.02 (case dimensions)

Weight = 17.1 K_g

Efficiency = 79.7 %

5. The structure of optimum design is developed based upon the requirements of a TYPE I design (page 4).

CHAPTER V
DESIGN OF DC PM MOTOR FOR THE ENHANCED TIME OF OPERATION

Previously, the structure of electric motor design was presented applicable to the case when the working machine to be equipped by a motor was already designed or existed. Now we will consider an important case when the design of the motor and the machine can be done simultaneously.

Based on the results of optimum design of a DC PM motor in Chapter IV, the structure of design will be modified to satisfy the requirements of enhanced time of operation. The time of operation can be reduced by two different ways: matching the load and actuator, and optimizing the velocity trajectory via proper design of a control system. This study will be focused on the time reduction by matching the load and actuator conditions presuming a possibility to provide a simulteneous design of mechanical and electrical parts.

We assume that the minimum time velocity trajectory is obtained (which is known to be a triangle [70]), and the load parameters such as load inertia, J_1, angular displacement, θ_1, are given as input information. The design variables of this system will be motor torque, T_r, and gear ratio. Under given torque, the inertia is determined from the optimum design of motor for minimum weight. The optimization of finding motor torque and gear ratio will reduce the operation time. If the weight of motor is to be minimized under each different value of motor torque, then this time reduction will require a big computer program and will take much computing time. However, if the motor inertia can be expressed as a function of motor torque from the result of weight minimization, then this problem will become much simpler. Thus, the hierarchical design structure of time minimization in actuator system is organized as shown in Fig. 5-1.

When the velocity trajectory and input parameters are given, the time of operation can be reduced as follows.

1. Find the functional relationship between the motor inertia and torque.
2. Find the motor torque and gear ratio that can reduce the operation time and can satisfy the maximum bending capacity of the weakest part.
3. Compute the maximum speed and maximum power in that given velocity trajectory.
4. Since the operating voltage is a function of magnetic flux that is determined in weight minimization, speed which was computed in step 3, and number of parallel paths in armature winding, the selection of terminal voltage will decide the diameter and the number of turns of armature winding.

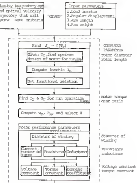

Velocity trajectory opt
Find optimal velocity trajectory that will improve some criteria
"GIVEN"
Input parameters
1.Load inertia
2.Angular displacement
3.Arm length
4.Arm weight
Find $J_m = f(T_r)$
Given T_r, find optimum design of motor for min N_t
Compute inertia J_m
Get functional relation
COMPUTED PARAMETER
rotor diameter
rotor length
Find T_r & G_r for min operation time
motor torque
gear ratio
Compute w_m, P_m, and select V
Motor performance parameters
Diameter of winding
diameter of winding
Current, number of turns
Resistance
Inductance
resistance
inductance
Voltage constant
Torque constant
Damping coefficient
voltage constant
torque constant
Design of control system

Fig. 5-1 Design structure of motor for actuator system

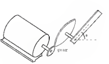
5. To determine the motor performance parameters, the following procedures are required.
   a. Compute the diameter of armature winding.
   b. Compute the number of turns, current, resistance, and inductance of armature winding.
   c. Compute the voltage constant and torque constant
6. The computed parameters of motor inertia, armature resistance, armature inductance, voltage constant, torque constant, and damping coefficient will be used as input parameters for the design of a control system which is out of the scope of this dissertation.

5.1 Formulation of Operation Time Enhancement

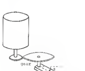
5.1.1 Objective Function

We consider a single motor actuating the first or the second joint of a multi-actuator system. We presume that for the goals of this research, all parameters of the system may be averaged. Weight minimization of the actuator at each link will have substantial effect, since the accelerating masses will be reduced correspondingly. Thus, minimizing the weight and time of operation for a single motor should lead to a substantial improvement in productivity for the whole manipulator.

Consider the motion of single joint of actuator-gear-load assembly as shown in Fig. 5-2. There are two basic



a. Raising arm motion



b. Waist rotation

Fig. 5-2 Schematic diagram of electric drive system

motions in this single joint—raising arm motion and waist motion. To raise the manipulator arm with load, the motor torque will cope with the weight of the arm, weight of the load, and acceleration of load and manipulator. The joint system will be formulated with a rigid body structure, and the bending capacity of gear teeth will limit the applied motor torque.

According to the D'Alembert's principle, the mechanism must be in equilibrium under the influence of the external and internal forces, so the following torque equilibrium equations can be obtained.

$$T_l = J_l \theta_l + B_l \dot{\theta}_l + T_w \tag{5.1}$$

$$T_m = J_m \theta_m + B_m \dot{\theta}_m + T_l' \tag{5.2}$$

where

T_l = load torque at load side

T_w = gravity torque of manipulator

$= W_t L \cos\theta$ (in waist motion $T_w = 0$) (5.3)

W_t = weight of manipulator

L = length of manipulator

T_l' - load torque at motor side

$= (J_l \theta_m + B_l \theta_m)/g_r^2 + T_w/g_r$ (5.4)

J_l = load inertia including manipulator inertia

J_m = motor inertia

B_m = damping coefficient at motor side

B_l = damping coefficient at load side

θ_m = angular displacement at motor side

θ_l = angular displacement at load side

q_r = gear ratio

From eq. (5.2) and eq. (5.4), we can get the following expression.

$$T_m = J_{eff}\ddot{\theta}_m + B_{eff}\dot{\theta}_m + T_w/q_r \tag{5.5}$$

where

J_{eff} = effective inertia at motor side

$= J_m + J_l/q_r^2$

B_{eff} = effective damping coefficient

$= B_m + B_l/q_r^2$

If we assume that this system is operating from a limited source of power and it is desired to have the system change from one state to another in minimum time, then it is necessary at all times to utilize all the power available, that is to use "bang-bang" control [70]. As a result of this control, the magnitudes of acceleration and deceleration are constant and maximum values, which will make the velocity trajectory triangular. From the triangular velocity trajectory, the acceleration torque, damping torque, load torque, and motor torque can be determined as in Fig 5-3. As can be seen from this figure, the motor torque which is the sum of acceleration torque, damping torque, and gravity torque is not a simple shape, it is a function of

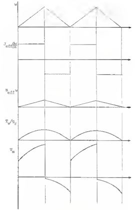


Fig. 5-3 Graphic representation of motor torque in a single joint system

angular displacement, speed, and acceleration. When we use the average torque, the analysis will become much simpler.

Rewriting eq. (5.4) and integrating yield

$$\int_0^{t_a} J_{eff}\alpha dt = \int_0^{t_a} T_m(t)dt - \int_0^{t_a} B_{eff}\alpha t dt - \int_0^{t_a} T_w/g_r dt \quad (5.6)$$

where

α = angular acceleration

The integration of motor torque will be simplified using the average rated torque, T_r.

$$\int_0^{t_a} T_m(t)dt = t_a T_r \qquad (5.7)$$

And the integration of gravity torque can be determined using the series expansion of cosine term:

$$\int_0^{t_a} T_w/g_r dt = \frac{1}{g_r}\int_0^{t_a} W_t L \operatorname{Cos}\left(\frac{t^2}{2g_r}\right)dt$$

$$= \frac{W_t L t_a}{g_r}\left[1 - \frac{(\Theta_1/2)^2}{10} + \frac{(\Theta_1/2)^4}{216} - \frac{(\Theta_1/2)^6}{9360} + - - - -\right] \qquad (6.8)$$

The average load torque is defined as

$$T'_{lavg} = T_{lavg}/g_r = \frac{1}{t_a}\int_0^{a} T_w g_r dt$$

$$= \frac{w_t L}{g_r}\left[1 - \frac{(\theta_1/2)^2}{10} + \frac{(\theta_1/2)^4}{216} - \frac{(\theta_1/2)^6}{9360} + \quad \right] \qquad (5.9)$$

Substituting eq. (5.7) - eq. (5.9) into eq. (5.6) yields

$$J_{eff}\omega_m = T_r t_a - \frac{\alpha t_a^2}{2} B_{eff} - T'_{1avg} t_a$$

$$(T_r - T'_{1avg})t_a^2 - \frac{g_r \theta_1 B_{eff}}{2} t_a - g_r \theta_1 J_{eff} = 0 \qquad (5.10)$$

The acceleration time can be determined by solving the quadratic equation of eq. (5.10).

$$t_a = \frac{g_r \theta_1 B_{eff} + \sqrt{(g_r \theta_1 B_{eff})^2 + 16 g_r \theta_1 J_{eff}(T_r' - T'_{1avg})}}{4(T_r - T'_{1avg})} \qquad (5.11)$$

The operation time is two times the acceleration time as can be seen from the symmetric triangular velocity trajectory.

$$t_0 = 2t_a \qquad (5.12)$$

In the waist motion, the gravity torque is zero, so eq. (5.6) becomes

$$\int_0^{t_a} J_{eff}\alpha dt = \int_0^{t_a} T_m(t) dt - \int_0^{t_a} B_{eff}\alpha t dt \qquad (5.13)$$

and the integration of damping torque is

$$T_b = \int_0^{t_a} B_{eff}\alpha t dt$$

$$= \frac{B_{eff}g_r\theta_1}{2} \qquad (5.14)$$

Substituting eq. (5.6) and eq. (5.14) into eq. (5.13) yields the following operation time.

$$t_o = \frac{B_{eff}g_r\theta_1 + \sqrt{(B_{eff}g_r\theta_1)^2 + 16\theta_1 J_{eff}g_r T_r}}{2T_r} \qquad (5.15)$$

The objective function for raising arm motion and waist motion are formulated as eq. (5.12) and eq. (5.15). The constraints in this time reduction problem is the bending limit of gear teeth. In this study, the spur gear will be used for the computation of gear bending capacity.

5.1.2 Bending Capacity of Spur Gear Teeth [71]

As the gear rotates, the number of pairs of gear teeth in contact simultaneously varies from one to two or more. However, the torque limit can be determined by imposing the most unfavorable conditions.

Consider the ability of a single tooth to carry bending load almost at the end of tooth as shown in Fig. 5-4. When the force F is applied at point A, the tangential component of F will produce a bending moment $F_b l$ at the base of the tooth. If we assume that the tooth is a cantilever beam, the bending limit of gear tooth will be determined as follows. The stress of a beam can be expressed as

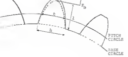

F_b
t
l
h
PITCH CIRCLE
BASE CIRCLE

Fig. 5-4 Bending capacity of a gear system

$$\sigma = \frac{6F_b l}{bh^2} = \frac{F_b}{b}\frac{6l}{h^2} \qquad (5.16)$$

where

b = thickness of tooth in axial direction

h = width of tooth

l = height of tooth

The factor $h^2/6l$ is a purely geometrical property of the size and shape of the tooth and may be written as a function of the circular pitch. The bending limit force of gear tooth will be expressed as

$$F_b = byp\sigma \qquad (5.17)$$

where

$y = h^2/(6lb)$ = Lewis factor

p = circular pitch = π/P_d

P_d = diametrical pitch

= number of teeth per inch (2.54 cm) of pitch diameter

Here, y is a pure number and is called Lewis factor which depends on the number of teeth in the gear and system of gearing used. The limit torque that can be applied to the gear tooth will be computed as

$$T_{limit} = F_b l$$

When the gear material, diametral pitch, and diameter of pitch circle are given, the bending capacity can be computed.

5.2 Time Reduction

Based on the procedure of time reduction illustrated in Fig. 5-1, the detailed analysis in each step will be discussed.

5.2.1 Functional Relationship

The functional relationships between inertia and torque can be obtained from the result of weight optimization in Chapter IV. The relations can be approximated using linear regression of the SAS package.

1. Normal structure of motor

$$J_m = 0.0023T_r^{1.1046}$$

2. 'Inside-out" structure of motor

$$J_m = 0.00023T_r^{1.2364}$$

These relations express the dependencies between the parameters when the motor is in optimum configuration with respect to minimum weight in normal and "inside-out" structure. Figure 5-5 shows the actual data and approximated data of inertia with different values of torque, which are quite close to each other.

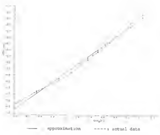


Fig. 5-5 Approximation of inertia as a function of motor torque and actual data from the result of weight minimization

5.2.2 Computation of T_r and g_r

Now the time minimum problem in raising arm motion is set as follows.

Minimize

$$t_o = \frac{g_r\theta_1 B_{eff} + \sqrt{(g_r\theta_1 B_{eff})^2 + 16 g_r\theta_1 J_{eff}(T_r' - T_{1avg}')}}{2(T_r - T_{1avg}')} \quad (5.18)$$

subject to maximum bending limit of gear teeth.

We will assume that the effective damping coefficient is zero; then eq. (5.18) of raising arm motion will be simplified as below:

$$\text{Minimize } t_o = \frac{2\sqrt{g_r\theta_1 J_{eff}(T_r - T_{1avg}')}}{T_r - T_{1avg}'} \quad (5.19)$$

subject to maximum bending limit

Among the variables in eq. (5.19), the load angular displacement and load inertia are given as input parameters. The parametric data for the PUMA robot, Unimate 500, are listed in Table 5-1. The motor inertia, J_m, was expressed as a function of motor torque.

When the constraint is not applied to this problem to see the general trend of time function, then the time function of eq. (5.19) can be plotted in three dimensions with motor torque and gear ratio as variables in Fig. 5-4. As can be seen from this figure, the operation time of the manipulator is reduced as the motor torque becomes larger.

Table 5-1 Parametric data for robot, UNIMATE 500

Load conditions		Specifications
Arm weight (kg)	W_t	54.08
Arm length (m)	L	0.432
Load inertia (kg-m^2)	J_1	10.08
Angular displacement (rad.)	θ_1	5.76

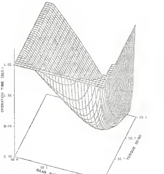


Fig. 5-6 Operation time function vs. gear ratio (g_r) and motor torque (T_r)

In a given motor torque, the optimum gear ratio can be found to have minimum operation time from this figure. The optimum gear ratio can be determined analytically as

$$\frac{\partial t_o}{\partial g_r} = 0 \rightarrow g_r = \sqrt{\frac{T_{1avg}}{T_r} + \left(\frac{T_{1avg}}{T_r}\right)^2 + \frac{J_1}{J_m}} \qquad (5.20)$$

So when the gear ratio in each motor torque is determined, then the operation time of raising arm motion can be drawn as shown in Fig. 5-7. We can see from this figure that the operation time of the manipulator using the 'inside-out' structure motor is 63% less than the manipulator using normal structure motor. When the maximum bending limit constraint is applied to this figure, the maximum available torque is the optimum torque and the gear ratio will be computed successively.

From eq. (5.17) and eq. (5.18), the bending limit torque can be computed with different values of diametral pitch and diameter of pitch circle as shown in Fig. 5-8. When the diametral pitch of the gear P_d = 10 and the diameter of the pitch circle D_{pc} = 2.5 inches (6.25 cm), the limit torque is determined from Fig. 5-8 to be 1.03 N-m. The operation time is determined from Fig. 5-7 to be 4.8 sec using normal structure of motor and 1.76 sec with "inside-out" structure of motor.

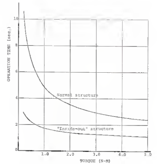


Fig. 5-7 Operation time vs. motor torque

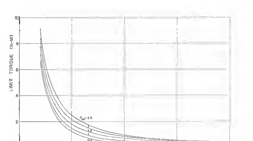


Fig. 5-8 Limit torque vs. diametral pitch(P_d) in different diameter of pitch circle(D_{pc})

5.2.3 Motor Performance Parameters

The performance parameters of the motor will be determined by the following procedures.

1. When the operation time is determined, the maximum speed and maximum power will be computed respectively as

$$w_m = \frac{2q_r\theta_1}{t_o} \qquad (5.21)$$

$$P_m = T_r w_m$$

2. If the terminal voltage is selected, then the diameter of winding that can deliver maximum power, the number of turns, current, resistance, and inductance [72] of armature winding can be computed as

$$D_w = \sqrt{\frac{4(T_r w_m + L_{cp})}{\pi J_a V}} \qquad (5.23)$$

$$N = mN_{as}\text{Integer}\left(\frac{A_a}{\pi D_w^2/4}\right) \qquad (5.24)$$

$$R = \frac{X_{1a}A_a N_{as} m}{(\pi a D_w^2/4)^2} \qquad (5.25)$$

$$L_a = \frac{\pi(N/a)^2 DL[N_p X_{1m}/(\pi D)]^3 \mu_0}{48 g_e} \qquad (5.26)$$

3. The voltage constant, K_e, and torque constant, K_t, will be computed as

$$K_e = \frac{N I_p \Phi}{\pi a} \tag{5.27}$$

$$K_t = \frac{T_r}{I_a} \tag{5.28}$$

5.3 Numerical Example

Using the Unimate 500, the design of a DC PM motor in a single joint motion is computed by those procedures with a result shown in Table 5-2. The "inside-out" structure motor has better characteristics in all areas.

5.4 Conclusions

Based on the analyses and the results of DC PM motor design for minimum time of operation, the following conclusions can be obtained:

1. The operation time is reduced by matching the load and the actuator conditions with maximum bending capacity of gear tooth, and the problem minimizing the weight and reducing the time of operation is solved via hierarchical design.
2. The operation time of raising arm motion using "inside-out" structure of motor is 63% less than that with normal structure of motor.

Table 5-2 Numerical example of motor design for robotic actuator

PARAMETERS		NORMAL STRUCTURE	"INSIDE-OUT" STRUCTURE
Torque	N-m	1.03	1.03
Rotor diameter	cm	6.835	4.306
Rotor length	cm	3.374	3.566
Armature slot area	cm^2	0.382	0.283
Magnet width	cm	7.189	4.542
Stator thickness	cm	0.868	0.872
Weight	kg	2.594	1.939
Inertia	$kg{-}m^2$	0.00226	0.00019
Operation time	sec.	4.8	1.76
Gear ratio		297	395
Maximum speed	RPM	5061	18698
Maximum power	HP	0.71	2.62
Diameter of armature coil	cm	0.088	0.151
Current	A	5.82	17.54
Number of armature coil		620	150
Resistance	Ω	1.97	0.163
Inductance	mH	12.414	0.425
Voltage constant		0.198	0.057
Torque constant		0.198	0.057

3. The structure of design is developed based upon the requirements of TYPE IIa design.

CHAPTER VI
MOTOR DESIGN STRUCTURES, CONCLUSIONS AND FUTURE WORK

Based upon the results of analyses in this dissertation this chapter will recommend the design structures of a motor for a multilink manipulator. The conclusions of this work are presented and suggestions for future work are discussed.

6.1 Design Structure

Figure 6-1 describes the structure of motor design for the actuators in a multilink manipulator. Actually, this design structure is the integration of discussions in Chapters IV and V. In other words, the operation time is reduced to find the most suitable motor torque after the first step optimization of motor design is finished. This first step design determines the values of main design variables. When the time reduction is finished, then the second step of motor design is done to find the detailed motor parameters and performance parameters such as diameter, current, number of turns in the armature winding, resistance and inductance of armature winding, and voltage and torque constant. The damping coefficient is hard to compute, so this may be obtained from measurement or from similar machines.

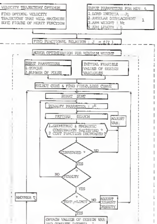

VELOCITY TRAJECTORY OPTIMUM
FIND OPTIMAL VELOCITY TRAJECTORY THAT WILL MAXIMIZE SOME FIGURE OF MERIT FUNCTION
INPUT PARAMETERS FOR MIN t
1 LOAD INERTIA J_1
2 ANGULAR DISPLACEMENT
3 ARM WEIGHT Wt
4 ARM LENGTH L
1
FIND FUNCTIONAL RELATION J = f(T)
MOTOR OPTIMIZATION FOR MINIMUM WEIGHT
INPUT PARAMETERS
1 TORQUE
2 NUMBER OF POLES
INITIAL FEASIBLE VALUES OF DESIGN VARIABLES
SELECT CORE & FIND FIELD,LOSS CURVE
START SUMT
PENALTY PARAMETER r^k
PATTERN SEARCH
ADJUST VAR.
1.GEOMETRIC & MAGNETIC CONSTRAINTS SATISFIED ?
2 COST FUNCTION DECREASED ?
CONVERGED ?
NO
YES
? PENALTY
NO
YES
ANOTHER T_i
? TEMP ≤LIMIT
NO
ADJUST CURRENT DENSITY
YES
OBTAIN VALUES OF DESIGN VAR AND COMPUTE INERTIA. J_m

Fig. 6-1 Structure of motor design for actuator

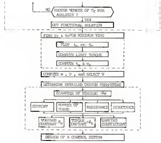

NO
ENOUGH NUMBER OF T_r FOR ANALYSIS ?
YES
GET FUNCTIONAL RELATION
FIND T_r & q_r FOR MINIMUM TIME
PLOT t_o vs T_r
COMPUTE LIMIT TORQUE
COMPUTE t_o & q_r
COMPUTE w , P , and SELECT V
DETERMINE DETAILED DESIGN PARAMETERS
DIAMETER OF WINDING D_w
CURRENT
NUMBER OF TURNS
RESISTANCE
INDUCTANCE
VOLTAGE CONSTANT K_e
TORQUE CONSTANT K_t
DAMPING COEFFICIENT
DESIGN OF A CONTROL SYSTEM

Fig. 6-1 - continued

6.2 Conclusions

The following conclusions may be obtained from this dissertation:

1. The study of the state of the art in motor design leads to the hierarchical optimum design structure which divides the optimization problem into several sub-optimization problems. This decoupled optimization will simplify the design noticeably and has the advantage of flexibility; detailed design that may be different from different purposes can be done using the same main program.
2. The notion "optimum configuration" is introduced which is defined as the length/diameter ratio leading to the minimization of the weight of the motor or maximization of its efficiency. The optimum configuration is computed theoretically for a number of motors. It is found that not all of the industrially manufactured motors satisfy the criterion of optimum configuration.
3. From the database we found that the DC PM motor has the highest T/W ratio among any other type of motor as shown in Table 6-1, and the best configuration of DC PM motor manufactured in industry is determined for the best T/W ratios or maximum efficiency as shown in Table 6-2.

Table 6-1 Comparison of T/W ratio between different types of motor

Type	Torque range			
	CL1	CL2	CL3	CL4
PM	0.058	0.145	0.249	0.422
Series	0.036	0.069	0.170	.
Shunt	0.046	0.077	.	0.147
Induction	0.050	0.113	0.206	0.303
Synchronous	0.030	0.061	.	.

Table 6-2 Comparison of optimum configurations

Criteria	Theory	Database
Minimum weight	0.89	1.3
Maximum efficiency	1.76	2.0

4. Most of the machines are proven to have their optimum configurations which may be different for different purposes, and the optimum configuration in weight minimization is not changed although the power rating is changed.
5. The result of optimum design of the DC PM motor shows that the DC PM motor has much better characteristics than the wound motor, and that the most desirable motor for the robotic applications or for the automated manufacturing system will be the DC PM "inside-out" motor with SmCo which will upgrade the performance characteristics while maintaining the technical requirements of the actuator.
6. The operation time is reduced by matching load and actuator conditions and the hierarchical structure of motor design is especially necessary to simplify the problems of multiple-goal optimization and to reduce the computing time.
7. The interactive computer programming that consists of 1200 lines of Fortran codes using the nonlinear programming algorithm, pattern search and Fibonacci search, can be applied to any other types of machine by just modifying the subprogram, "DESIGN," which differs for different types of motors.
8. The result of optimum motor design can be applied to the actual manufacturing of the motor and will improve the motor cost effectiveness and other

characteristics while increasing the productivity of robotic applications.

6.3 Suggestions for Future Research

1. In the analysis of the database, the dimensions of the motor are the outside case dimensions which are proportional to the rotor dimensions. If the exact rotor dimensions can be obtained from the industry, we can suggest a more accurate optimum configuration of the motor for minimum weight and maximum efficiency.
2. If the motor is actually manufactured from the result of optimum design and all the performance parameters and temperature rise are measured from this manufactured motor, the optimum design of the motor can be further improved by comparing the computed parameter with the measured parameters.
3. The operation time was reduced by matching the load and actuator conditions with the concept of an average torque. The exact analysis of load and motor torque will provide more accurate results.

APPENDIX A
TRANSFORMER

Figure 4-2 shows the simplified geometry of C - core transformer. If the flux varies sinusoidally with time the primary and secondary induced voltages are

$$E_k = 4.44\, N_k\, f\, \phi_m \qquad \text{(A.1)}$$

where

E_1, E_2 = rms primary and secondary voltages
N_1, N_2 = number of primary and secondary windings
f = frequency of power source
ϕ_m = peak value of magnetic flux

The window area which is the winding area of transformer is assumed to be divided equally between primary and secondary windings, therefore the total number of turns in the primary winding can be determined as

$$N_1 = \frac{2L_c(D_c - D_m)k_u}{(\pi D_w^2/4)} \qquad \text{(A 2)}$$

where

k_u = window utilisation factor
D_w = diameter of wire

In order to calculate the current in any winding, we considered the current density at first. The losses of a transformer is proportional to the cube of geometrical dimension and the square of current density [73]

$$P_{loss} = KL^3 J^2 \tag{A.3}$$

The area product which is an important variable for relating the power losses, the dimension, and the number of windings, can be expressed as,

$$\begin{aligned} A_p &= (\text{window area}) \times (\text{cross section area}) \\ &= L_c D_m^{\ 2} (D_c - Dm)/2 \end{aligned} \tag{A.4}$$

Then the losses can be expressed as,

$$P_{loss} = K' A_p^{\ 0.75} J^2 \tag{A.5}$$

The average temperature increase of a transformer winding is proportional to the energy losses and inversely proportional to the surface area of transformer. As a result the temperature rise may be written as,

$$\Delta T = \frac{K_2 A_s^{\ 0.75} J^2}{K_3 A_p^{\ 0.5}} \tag{A.6}$$

Thus at a given temperature rise the current density is expressed as a function of area product.

$$J = K_j \left[\frac{2}{L_c D_m^{\ 2} (D_c - D_m)} \right]^{0.125} \tag{A.7}$$

The induced output volt-ampere(VA) is given by

$$\begin{aligned} (VA) &= E_2 J_2 = 4.44 N_2 f D_m^{\ 2} B_m I_2 \\ &= K L_c^{\ 0.875} (D_c - D_m)^{0.875} D_m^{\ 1.75} \end{aligned} \tag{A.8}$$

where

$$K = 2.42\ k_u f B_m K_j$$

and the losses of transformer can be determined as follows. Mean turn length of each winding is

$$M_{1t} = 2(D_c + D_m) \qquad (A.9)$$

The total length in primary and secondary windings becomes

$$T_1 = M_{1t}N = \frac{L_c(D_c + D_m)(D_c - D_m)k_u}{(\pi D_w{}^2/4)} \qquad (A.10)$$

Resistance of each winding is then

$$R = \frac{4\rho L_c(D_c + D_m)(D_c - D_m)k_u}{(\pi D_w{}^2/4)^2} \qquad (A.11)$$

And the copper losses in each winding can be expressed as

$$L_{cp} = \frac{\rho L_c(D_c + D_m)(D_c - D_m)^{0.75}k_u K_J{}^2 2^{0.25}}{(L_c D_m{}^2)^{0.25}} \qquad (A.12)$$

When the maximum magnetic flux density is decided we can assume that the core losses per unit weight are constant.

$$L_{ir} = L'_{ir} W_{ir} \qquad (A.13)$$

where

L'_{ir} = core losses per unit weight

APPENDIX B
SYNCHRONOUS MOTOR WITH PM ROTOR

When the permanent magnet is used as a field excitation source as in Fig. 4-5, the reluctance of the PM becomes not negligible in field computation for this kind of structure. Therefore an accurate mathematical model in field determination must be employed to take all the parameters into account. When the motor is operated at synchronous speed there are no free currents inside the stator. As a result, the flux pattern in a steady state is not changed, which enables us to solve the Laplace equation.

The governing equations for this case are

$$\nabla \cdot \bar{B} = 0 \tag{B.1}$$

$$\nabla \times \bar{H} = 0 \tag{B.2}$$

Because the curl of $\bar{H}$ is zero, a scalar magnetic potential can be defined as,

$$\bar{H} = -\nabla \psi \tag{B.3}$$

From equations (B.1) and (B.2) with $\bar{B} = \mu \bar{H}$, the magnetic scalar potential obeys the Laplace equation

$$\nabla^2 \psi = 0 \tag{B.4}$$

In this analysis the variation of scalar potential along the z-axis is neglected. And if only the fundamental component of harmonics of stator winding current is considered, the three phase windings approximates a surface current distribution of the form

$$\bar{K}(\phi,t) = \mathrm{Re}\, K_z e^{j(wt-m\phi)}\, \hat{a}_z \tag{B.5}$$

where

$K_z = N_{ap} i_a$

m = number of pole pairs

The magnetic potential within the stator is then also sinusoidally varying with time.

$$\Psi(r,t) = \mathrm{Re}\, \hat{\Psi}(r) e^{j(wt-m\phi)} \tag{B.6}$$

Substituting eq.(B.6) into eq.(B.4) with cylindrical coordinates yields

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\Psi(r)}{dr}\right) - \frac{m^2}{r^2}\Psi(r) = 0 \tag{B.7}$$

The magnetic scalar potential in different regions such as rotor, airgap, stator, and outside the motor can be expressed accordingly

1. Rotor ; $\hat{\Psi}_1(r) = A_1 r^m \qquad r \le a_1$ (B.8)

2. Airgap : $\hat{\Psi}_2(r) = A_2 r^m + A_3/r^m$; $a_1 \le r \le a_2$ (B.9)

3. Stator : $\hat{\Psi}_3(r) = A_4 r^m + A_5/r^m$; $a_2 \le r \le a_3$ (B.10)

4. Outside motor : $\hat{\Psi}_4(r) = A_6/r^m$; $r \ge a_3$ (B.11)

The above magnetic scalar potential in different regions can be obtained from eq.(B.8)-eq.(B.11) by applying the boundary conditions at each point as follows

1. $r = a_1$: $H_{1\phi} = H_2$, $B_{1r} = B_{2r}$ (B.12)

2. $r = a_2$: $H_{2\phi} - H_{1\phi} = K_z$, $B_{2r} = B_{3r}$ (B.13)

3. $r = a_3$. $H_{3\phi} = H_{4\phi}$, $B_{3r} = B_{4r}$ (B.14)

The magnetic scalar potential at each region is

$$\Psi_1(r,\phi,t) = (jX_1 K_z r^m + X_1' r^m e^{-j\delta})\, e^{j(wt-m\phi)} \qquad (B.15)$$

$$\Psi_2(r,\phi,t) = [jK_z(X_2 r^m + \frac{X_3}{r^m}) + e^{j\delta}(X_2' r^m + \frac{X_3}{r^m}]\, e^{j(wt-m\phi)} \qquad (B.16)$$

$$\Psi_3(r,\phi,t) = [jK_z(X_4 r^m + \frac{X_5}{r^m}) + e^{j\delta}(X_4' r^m + \frac{X_5'}{r^m})]\, e^{j(wt-m\phi)} \qquad (B.17)$$

$$\Psi_4(r,\phi,t) = (jK_z X_6/r^m + e^{j\delta} X_6'/r^m)\, e^{j(wt-m\phi)} \qquad (B.18)$$

The magnetic field in each region can be obtained from the gradient of magnetic scalar potential as in next page.

$$H_1=\left[X_1K_z m r^{m-1}\mathrm{Sin}(wt-m\phi)-X_1' m r^{m-1}\mathrm{Cos}(wt-m\phi-\delta)\right]\hat{a}_r$$
$$+\left[-m r^{m-1}X_1K_z\mathrm{Cos}(wt-m\phi)-X_1' m r^{m-1}\mathrm{Sin}(wt-m\phi-\delta)\right]\hat{a}_\phi \qquad (B.19)$$

$$H_2=\left[m\left(X_2r^{m-1}-\frac{X_3}{r^{m-1}}\right)K_z\mathrm{Sin}(wt-m\phi)-m\left(X_2'r^{m-1}-\frac{X_3'}{r^{m-1}}\right)\mathrm{Cos}(wt-m\phi-\delta)\right]\hat{a}_r$$
$$+\left[-m\left(X_2r^{m-1}+\frac{X_3}{r^{m+1}}\right)K_z\mathrm{Cos}(wt-m\phi)-m\left(X_2'r^{m-1}+\frac{X_3'}{r^{m+1}}\right)\mathrm{Sin}(wt-m\phi-\delta)\right]\hat{a}_\phi \qquad (B.20)$$

$$H_3=\left[m\left(X_4r^{m-1}-\frac{X_5}{r^{m+1}}\right)K_z\mathrm{Sin}(wt-m\phi)-m\left(X_4'r^{m-1}-\frac{X_5'}{r^{m+1}}\right)\mathrm{Cos}(wt-m\phi-\delta)\right]\hat{a}_r$$
$$+\left[-m\left(X_4r^{m-1}+\frac{X_5}{r^{m+1}}\right)K_z\mathrm{Cos}(wt-m\phi)-m\left(X_4'r^{m-1}+\frac{X_5}{r^{m+1}}\right)\mathrm{Sin}(wt-m\phi-\delta)\right]\hat{a}_\phi \qquad (B.21)$$

$$H_4=\left[-\frac{X_6K_z}{r^{m+1}}\mathrm{Sin}(wt-m\phi)+\frac{mX_6'}{r^{m+1}}\mathrm{Cos}(wt-m\phi-\delta)\right]\hat{a}_r$$
$$+m\left[-\frac{X_6}{r^{m+1}}K_z\mathrm{Cos}(wt-m\phi)-\frac{X_6'}{r^{m+1}}\mathrm{Sin}(wt-m\phi-\delta)\right]\hat{a}_\phi \qquad (B.22)$$

The expressions of $X_1 - X_6$ and $X_1' - X_6'$ are as follows. The order of variables is not in numeric order because the derivations can start from different variables

$$X_3=\frac{Y_3}{K_3'Y_1+Y_2}$$

$$X_3'=-\frac{K_3''Y_1}{K_3'Y_1+Y_2}$$

$$X_2=K_3'X_3$$

$$X_2'=K_3'X_3'+K_3''$$

$$X_1 = X_2 + X_3/a_1^{2m}$$

$$X_1' = X_2' + X_3'/a_1^{2m}$$

$$X_5 = 0.5\left[a_2^{2m}X_2(1-\mu_e/\mu)+(1+\mu_e/\mu)X_3+a_2^{m+1}/m\right]$$

$$X_5' = 0.5\left[a_2^{2m}X_2'(1-\mu_e/\mu)+(1+\mu_e/\mu)X_3'\right]$$

$$X_4 = X_5K_4'$$

$$X_4' = X_5'K_4'$$

$$X_6 = a_3^{2m}X_4 + X_5$$

$$X_6' = a_3^{2m}X_4' + X_5'$$

where

$$K_4' = (\mu-\mu_e)/(\mu+\mu_e)a_2^{2m}$$

$$K_3' = \mu_e M/\ (\mu-\mu_e)ma_1^{m-1}$$

$$K_3'' = -(\mu+\mu_e)/\ (\mu-\mu_e)ma_1^{2m}$$

$$Y_1 = \frac{1+\mu_e/\mu}{a_2^{2m}} - K_4'(1-\mu_e/\mu)$$

$$Y_2 = \frac{1}{a_2^{2m}}\left[\frac{1-\mu_0/\mu}{a_2^{2m}} - X_4'(1+\mu_0/\mu)\right]$$

$$Y_3 = \frac{1}{ma_2^{m-1}}(X_4' - 1/a_2^{2m})$$

$$a_1 = d/2$$

$$a_2 = a_1 + g$$

$$a_3 = a_2 + T_f$$

Among those magnetic fields only the radial component of magnetic field in the airgap, B_{2r}, and the tangential component of magnetic field in the stator are of interest to us. The magnetic flux in the stator is obtained from the integration.

$$\begin{aligned}\Phi_{3\phi} &= \mu\int\int H_{3\phi}\,dr\,dL \\ &= \mu L\{X_5(1/a_3^m - 1/a_2^m) - X_4'(a_3^m - a_2^m)\ K_z \operatorname{Cos}(wt-m\phi) \\ &\quad + X_5'(1/a_3^m - 1/a_2^m) - X_4'(a_3^m - a_2^m)\ \operatorname{Sin}(wt-m\phi-\delta)\}\end{aligned} \tag{B.23}$$

The magnetic flux linkage that is cut by the stator winding coil is

$$\lambda = \frac{\pi\mu_0 L K_w N}{2}(X_3'/r^m - X_2' r^m)\ \operatorname{Cos}\phi \tag{B.24}$$

The coenergy stored in the airgap is

$$W' = \frac{\pi\mu_o LK_W Ni}{2}(X_3'-X_2'r^m) \text{ Cos } \delta \tag{B.25}$$

The torque exerted on the PM rotor becomes

$$T = -0.5\pi\mu_o LK_W N_1(X_3' - X_2'r^m) \text{ Sin } \delta \tag{B.26}$$

The pullout torque and the average torque of a three phase m pole pair motor are then

$$T_{po} = 3.332m\mu_o LK_W(X_3'-X_2'r^m)Ni \tag{B.27}$$

$$T_{av} = -\mu_o LK_W Ni(X_3'-X_2'r^m) \text{ Sin } \delta \tag{B.28}$$

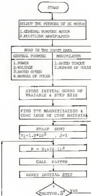
When the number of pole is selected the angular speed of rotor in a synchronous motor is determined and the average output power can be computed accordingly as,

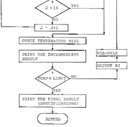
$$P_{av} = T_{av}\, w \tag{B.29}$$

APPENDIX C

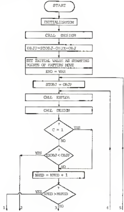
FLOW CHART OF COMPUTER PROGRAM

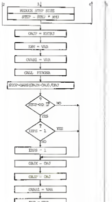
C.1 Main program

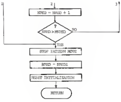

START
SELECT THE PURPOSE OF DC MOTOR
1. GENERAL PURPOSE MOTOR
2. MULTILINK MANIPULATOR
READ IN THE INPUT DATA
GENERAL PURPOSE
MANIPULATOR
1. POWER
2. VOLTAGE
3. RATED SPEED
4. NUMBER OF POLES
1. RATED TORQUE
2. NUMBER OF POLES
STORE INITIAL GUESS OF VARIABLE & STEP SIZE
FIND THE MAGNETIZATION & CORE LOSS OF CORE MATERIAL
START SUMT
R1=1.0*10^5 , J=1
R = R1x(0.1)^J
CALL PATTERN
RESET INITIAL STEP
?
PNLTY≤0.D^6
YES
NO
1
2
3
4


1
2
3
4
?
J = 15
YES
NO
J - J+1
CHECK TEMPERATURE RISE
PRINT THE INTERMEDIATE RESULT
VAR=OVAR
ADJUST XJ
?
TEMP ≤ LIMIT
NO
YES
PRINT THE FINAL RESULT (SPECIFICATIONS)
RETURN

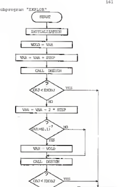
C 2 Subprogram "PATTRN"


START
INITIALIZATION
CALL DESIGN
OBJP=STOBJ-OBJX=OBJ
SET INITIAL VALUE AS STARTING POINTS OF PATTERN MOVE
EXO = VAR
STOBJ = OBJP
CALL EXPLOR
CALL DESIGN
?
C = 1
YES
NO
?
EXOBJ < OBJP
YES
NO
NRED = NRED + 1
?
NRED > MNRED
YES
NO
1
2
3
4
5

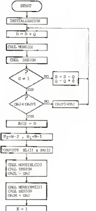

1
2
3
4
5
REDUCE STEP SIZE
STEP = STEP * XMU
OBJP = EXOBJ
EXN = VAR
OVAR1 = VAR
CALL FIBONA
STOP=DABS(OBJX-OBJ)/OBJ
?
STOP<(0 1)
NO
YES
?
IEPS = 1
YES
NO
IEPS = 1
OBJX = OBJ
OBJP = OBJ
OVAR1 = VAR
EXO = VAR
2

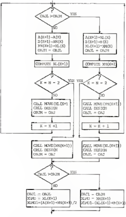

1
2
3
NRED = NRED + 1
?
NRED > MNRED
NO
YES
STOP PATTERN MOVE
NRED = NRED1
RESET INITIALIZATION
RETURN

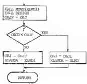
3 Subprogram "EXPLOR"


START
INITIALIZATION
VOLD = VAR
VAR = VAR + STEP
CALL DESIGN
?
OBJ < EXOBJ
YES
NO
VAR = VAR - 2 * STEP
?
VAR>(0.1)^-7
NO
YES
VAR = VOLD
CALL DESIGN
?
OBJ < EXOBJ
YES
NO
EXOBJ = OBJ
VAR = VOLD
RETURN

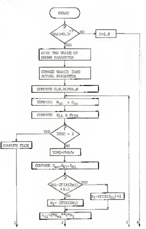
4 Subprogram 'FIBONA'

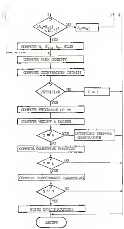

START
INITIALIZATION
D = D + Q
CALL MOVE(D)
CALL DESIGN
?
C ≠ 1
NO
D = D - Q
Q = Q * 0
YES
?
OBJ < OBJP1
NO
OBJP1=OBJ
YES
B(1) = D
N2=N-2 , N1=N-1
COMPUTE XL(1) & XM(1)
CALL MOVE(XL(1))
CALL DESIGN
OBJL = OBJ
CALL MOVE(XM(1))
CALL DESIGN
OBJM = OBJ
K = 1


?
OBJL >OBJM
YES
NO
A(K+1)=A(K)
B(K+1)=XM(K)
XM(K+1)=XL(K)
OBJM = OBJL
A(K+1)=XL(K)
B(K+1)=B(K)
XL(K+1)=XM(K)
OBJL = OBJM
COMPUTE XL(K+1)
COMPUTE XM(K+1)
?
K = N - 2
YES
YES
?
K = N - 2
NO
NO
CALL MOVE(XL(K+1
CALL DESIGN
OBJM = OBJ
CALL MOVE(XM(K+1))
CALL DESIGN
OBJL = OBJ
K = K +1
K = K + 1
CALL MOVE(XM(K+1))
CALL DESIGN
OBJM = OBJ
CALL MOVE(XL(K+1))
CALL DESIGN
OBJL = OBJ
?
OBJL >OBJM
YES
NO
OBJ1 = OBJL
XLMD = XL(K+1)
XLMD1=(A(K+1)+XM(K+1))/2
OBJ1 = OBJM
XLMD = XM(K+1)
XLMD1=(XL(K+1)+B(K+1))/2

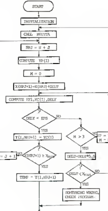

CALL MOVE(XLMD1)
CALL DESIGN
OBJF = OBJ
?
OBJ1 < OBJF
YES
NO
OBJ = OBJF
XLAMDA = XLMD1
OBJ = OBJ1
XLAMDA = XLMD
RETURN

c) Subprogram "DESIGN"


START
?
VAR>(0.1D^-7
NO
C=1.0
YES
GIVE THE VALUE OF KNOWN PARAMETER
CHANGE VAR(I) INTO ACTUAL PARAMETER
COMPUTE G,W,ALPHA,M
COMPUTE W_BS & D_AS
COMPUTE X_LA & X_I2R
?
ISSC = 2
YES
COMPUTE FLUX
NO
TORQ=PWR/W
COMPUTE D_WR, X_N7, Z_N1
?
ZN1-IFIX(ZN1) ≥ 0.5
YES
N_1=IFIX(Z_N1)+1
NO
N_1= IFIX(ZN_1)
A_A1=X*D_WR**2*N_1
1
2
3
4


2
3
4
1
?
|A_n-A_n1| < (0.1)^-6
NO
A_n=A_n1
YES
COMPUTE N, X_1m, A_m, FLUX
COMPUTE FLUX DENSITY
COMPUTE CONSTRAINTS CNTR(I)
?
CNTR(I)<0
NO
C = 1
YES
COMPUTE THICKNESS OF PM
COMPUTE WEIGHT & LOSSES
?
Y = 4
YES
DETERMINE THERMAL CONSTRAINTS
NO
COMPUTE OBJECTIVE FUNCTION
?
Y = 1
NO
YES
COMPUTE PERFORMANCE PARAMETERS
?
Y = 7
NO
YES
PRINT SPECIFICATIONS
RETURN

C 6 Subprogram "THERML"


START
INITIALIZATION
CALL RKUTTA
NRJ = N + J
COMPUTE YP(I)
M = 0
X(NRJ+1)=X(NRJ)+DELT
COMPUTE FP1,YC(I),DELY
?
DELY < EPS
NO
YES
Y(I,NRJ+1) = YC(I)
?
M > 3
NO
YP = YC
M = M+1
YES
DELT=DELT*0.5
J = J + 1
NO
?
X(NRJ+1) ≥ X_END
YES
?
DELT < X_start
NO
YES
TEMP = Y(1,NRJ+1)
SOMETHING WRONG,
CHECK PROGRAM.
RETURN

APPENDIX D
FLOW CHART OF COMPUTER PROGRAMS

For the execution of this computer program the Harris 800 system was used. The procedures to access to this computer program are as follows:

1 Sign on
   - a. Turn on the terminal.
   - b. Hit <control-G> Prompt (USER # ?)
   - c. Enter user name <space><control-Z> password <return>
2. Compiling the computer program "OPTIM"
   - a. Free all <return>
   - b. SAUF77 OPTIM <return>
   - c. VU.R <return>
   - d. MO NX <return>
   - e. Begin <return>
3. Assigning each file to each storage
   - a. AS 15=OPTDAT <return>
   - b. AS 1=APPR <return>
   - c. AS 2=SPEC <return>
   - d. AS 4=EVE <return>
   - e. AS 6=* <return>
   - f. AS 7=STEP <return>
   - g. AS 8=* <return>
   - h. AS 9=ZZ <return>
4. Execution
   - a. XE <return>

```
C***********************************************************************
C
C  ** THE MAIN PROGRAM WILL DO THE FOLLOWING JOBS.
C   A.DECIDES THE USAGE OF MOTOR TO BE DESIGNED
C   B.READS IN THE INPUT DATA
C   C.CONTROLS THE SEQUENTIAL UNCONSTRAINED MINIMIZATION TECHNIQUE (SUMT)
C     BY ADJUSTING THE PENALTY FACTOR
C   D.ASSIGNS THE NUMBER OF VARIABLES. STEP REDUCTION RATIO. MAXIMUM
C     NUMBER OF STEP REDUCTIONS
C   E.GIVES THE INITIAL FEASIBLE SET OF SOLUTIONS
C
C  ** THE IMPORTANT VARIABLES USED IN THIS PROGRAM ARE;
C   A NVAR : NUMBER OF VARIABLES
C   B.XMU : STEP REDUCTION RATIO
C   C.MNRED : MAXIMUM NUMBER OF STEP REDUCTIONS
C   D.VAR(I) : DESIGN VARIABLES
C   E.STEP(I) : DISTANCE OF SEARCHING MOVEMENT IN EACH DESIGN VARIABLES
C   F.OSTEP(I) . INITIAL STEP
C   G.OBJ : THE VALUE OF OBJECTIVE FUNCTION
C   H.R : PENALTY PARAMETER
C
C***********************************************************************
C
C
      IMPLICIT DOUBLE PRECISION*6 (A-H,O-Z)
      DIMENSION VAR(6),OVAR(6),STEP(6),OSTEP(6),A(15),CH(11),CL(8)
      COMMON /COM1/VAR,XJ/COM2/STEP/COM3/NVAR
     1       /PATN/MNRED,XMU/OUTP/J/RESL/TEMP,NADJ
     2       /OBJE/P,NT,PNLT/OBJ2/OBJ/CTRL/NRED1
     3       /CHEB/A/HFLD/CH/LOSS/CL/SELE/ISEC,PWR,V,NRPM,IP,TORQ
C
C  ** START THE OPTIMIZATION PROGRAM
C
```

```
      WRITE(6,1)
1     FORMAT(' THIS COMPUTER PROGRAM WILL DETERMINE THE MOTOR DIMENSIONS
     1 FOR',/,'   THE OPTIMUM DESIGN TO HAVE MINIMUM WEIGHT.',/,
     2' PLEASE ANSWER THE FOLLOWING QUESTIONS.')
      WRITE (6,2)
2     FORMAT(' WHAT IS THE PURPOSE OF THIS MOTOR YOU WANT TO DESIGN ?
     1',/,'  A.IF,FOR GENERAL PURPOSE MOTOR,THEN TYPE 1
     2',/,'  B.IF,FOR MULTI-LINK MANIPULATOR,THEN TYPE 2,')
      READ (8,3) ISEC
3     FORMAT(I2)
      WRITE (6,4)
4     FORMAT(' 1,ENTER THE INPUT PARAMETER OF MOTOR.')
      IF (ISEC.EQ.1) GO TO 7
      IF (ISEC.EQ.2) GO TO 8
C
7     WRITE (6,11)
11    FORMAT('  A, OUTPUT POWER IN HP = ?')
      READ (8,12) PWR1
12    FORMAT(F3.0)
      PWR=PWR1*746.0D0
      WRITE (6,13)
13    FORMAT('  B. OPERATING VOLTAGE = ?')
      READ (8,14) V
14    FORMAT(F10.0)
      WRITE (6,15)
15    FORMAT('  C, RATED SPEED IN RPM = ?')
      READ (8,16) NRPM
16    FORMAT(I4)
      WRITE (6,17)
17    FORMAT('  D, NUMBER OF POLES = ?')
      READ (8,18) IP
18    FORMAT(I2)
      GO TO 19
C
```

```
5       WRITE(6,2I)
21      FORMAT('  A.RATED TORQUE IN NT-M = ?')
        READ (8,22) TORQ
22      FORMAT(F3.1)
        WRITE (6,23)
23      FORMAT('  B. NUMBER OF POLES = ?')
        READ (8,24) IP
24      FORMAT(I2)
        WRITE (6,25)
25      FORMAT('  C. TERMINAL VOLTAGE = ?')
        READ (8,26) V
26      FORMAT(F10.0)
        WRITE (6,27)
27      FORMAT('  D, MAXIMUM SPEED IN RPM = ?')
        READ (8,28) NRPM
28      FORMAT(I4)
C
17      WRITE (6,I4I)
I4I     FORMAT('  E. CURRENT DENSITY IN A/CM**2 = ?')
        READ (8,29) XJI
29      FORMAT(F10.0)
C
C  ** ASSIGN THE VALUES OF INITIAL FEASIBLE SOLUTION AND CONTROL PARAMETERS
C
        READ (15,10) NVAR,XMU,MNRED
10      FORMAT(I2,D4.1,I3)
        READ (15,20) (VAR(I),I=I,NVAR)
20      FORMAT(2D7,5,D9,7,2D7,5)
        READ (15,30) (STEP(I),I=I,NVAR)
30      FORMAT(2D6,4,D8.5,2D6,4)
C
C  ** STORE THE INITIAL STEP
C
        DO 40 I=I,NVAR
        OSTEP(I)=STEP(I)
        OVAR(I)=VAR(I)
40      CONTINUE
```

```
C
C  ** WRITE THE STARTING POINTS & INITIAL STEPS
C
      WRITE (7,35) (VAR(I),I=1,NVAR),(STEP(I),I-1,NVAR)
35    FORMAT(' INITIAL VALUE  VAR(I)-',5D14,5//10X,
     1'STEP(I)=',5D14,5///)
C
C  ** PRINT THE HEADING
C
      WRITE (7 61)
61    FORMAT('0 ,' J',5X,'R' 9X,'NREO' 10X,'O' 11X,'L',11X,'AA',10X,'LM',
     110X 'TF',10X,'TM',9X,'DWA' 10X,'TH')
      WRITE (7,62)
62    FORMAT(5X,'OBJ',15X,'WT',14X,'EFF',10X,'EG',9X,'FLUX',9X,'BM',10X,
     1'PT',10X,'BAY',9X,'BFY')
C
C  ** COMPUTE THE COEFFICIENT OF CHEBYSHEV POLYNOMIALS TO FIND MAGNETIC
C     FIELD & LOSS OF SOFT MAGNETIC MATERIAL
C
C   * COMPUTE THE COEFFICIENT OF H=F(B)
C
      CALL CHEBY
      DO 75 I=1,11
      WRITE (1,47) I,A(I)
47    FORMAT(' CH(',I2,')',D10.3)
75    CH(I)=A(I)
C
C   * COMPUTE THE COEFFICIENT OF LOSS=F(B)
C
      CALL CHEBY
      DO 85 I=1,9
      WRITE(1,48) I,A(I)
48    FORMAT(' CL(',I2,')',D10.3)
85    CL(I)-A(I)
C
32    XJ=(J1+1.0D4
C
C  **ASSIGN THE VALUES OF PENALTY PARAMETERS AND EXECUTE 'SUMT'
C
      J=I
      RI=I.0D5
70    R=RI*0.ID0**J
      WRITE (4,45) J,R
45    FORMAT('  J=',I3,5X,'R=',D14.5)
      CALL PATTRN
      Y=1.0D0
      CALL DESIGN(Y)
      Y=0.0D0
      DO 80 I=I,NVAR
      STEP(I)=OSTEP(I)
80    CONTINUE
C
C  ** CHECK THE SATISFACTORY LEVEL OF CONVERGENCE
C     IF THE CONVERGENCE CRITERION IN EACH ITERATION IS
C     SATISFIED, STOP OPTIMIZATION AND PRINT THE RESULT.
C
      IF((OBJ-PNLT).LT.0.1D-6) GO TO 90
      IF(J.GT.20) GO TO 90
      J=J+I
      GO TO 70
90    Y=4.0D0
      CALL DESIGN(Y)
C
      WRITE(6,I7I)
171   FORMAT(' THE RESULT OF INTERMEDIATE OPTIMIZATION IS:')
      DO 172 I=1,NVAR
      WRITE (6,173) I,VAR(I)
173   FORMAT('0','    VAR(',I2,')= ',DI2.5)
172   CONTINUE
      WRITE (6,I74) TEMP
174   FORMAT('0','** THE MAXIMUM TEMPERATURE RISE IS',DI2.5)
      WRITE(6,175)
```

```
175   FORMAT(' DO YOU WANT TO ADJUST TEMPERATURE RISE ?',/,
     1'    IF TOU WANT,TYPE 1',/,
     2'    IF NOT,TYPE 0')
      READ (8,176) NADJ
176   FORMAT(I2)
      IF (NADJ,EQ,0) GO TO 497
      WRITE (6,177) XJ,TEMP
177   FORMAT('0',' THE CURRENT DENSITY WAS' D12,5,/
     1' AND THE TEMPERATURE RISE WAS' D12,5,/,
     2' NOW WHAT WILL BE THE CURRENT DENSITY ?')
      READ (8,178) XJI
178   FORMAT(F5,1)
      Y=0,0D0
      DO 179 I=I,NVAR
      VAR(I)=DVAR(I)
179   CONTINUE
      WRITE(6,765) NRPM-IP PWR,V,XJI (VAR(I),I-I NVAR)
765   FORMAT(I8,I4,8(D10.3,1X))
      GO TO 32
C
497   Y-7,0
      CALL DESIGN(Y)
      WRITE (6,1234)
1234  FORMAT(' THE OPTIMIZATION PROCEDURE IS FINISHED')
      STOP
      END
C
C
C
      SUBROUTINE CHEBY
C
C  ** THIS SUPPROGRAM 'CHEBY' WILL FIND THE COEFFICIENT OF CHEBYSHEV
C     POLYNOMIALS FROM THE EQUIDIATANT DATA
C
      IMPLICIT DOUBLE PRECISION*6 (A-H,O-Z)
      COMMON /CHEB/Y
      DIMENSION R(25),V(100),Y(25),C(25) F(25)
```

```
C
C   ** COMPUTE ROOTS OF XPAR
C
       READ (15,10) M,N,XMIN,DELTX
10     FORMAT(2I2,2D3.1)
       M=M+1
       X2=M
       DO 20 I=1,M
       X1=I
       ARG=1.570796326B*(2.*X1-1)/X2
       MSUB=M+1-I
       R(MSUB)=DCOS(ARG)
20     CONTINUE
C
C   ** NORMALIZE VECTORS
C
       X1=N-1
       DV=2.0/X1
       V(1)=-1.0
       L=N-1
C
       DO 30 I=1,L
       V(I+1)=V(I)+DV
30     CONTINUE
C
       READ (15,40) (Y(J),J=1,N)
40     FORMAT(8F8.3,/,8F8.3,/,3F8.3)
C
C   ** PERFORM LAGRANGIAN INTERPOLATION
C
       I=1
       DO 50 L=1,N
       IF(R(I).GT.V(L)) GO TO 50
51     U=(R(I)-V(L-1))/(V(L)-V(L-1))
       IF (L.GT.2) GO TO 55
54     F(I)=U*(Y(L)-Y(L-1))+Y(L-1)
```

```
      GO TO 57
55    IF (L.GE.N) GO TO 54
54    ZIP=-U*(U-1.)*(U-2.)*Y(L-2)/6.+(U*U-1.)*(U-2)*Y(L-1)/2.
      F(I)=ZIP-(U+1.)*(U-2.)*U*Y(L)/2.+U*(U*U-1.)*Y(L+1)/6.
57    I=I+1
      IF ((M+1).LE.I) GO TO 53
      IF (R(I).LE.V(L)) GO TO 51
50    CONTINUE
C
C  ** COMPUTE COEFFICIENTS
C
53    ZM=M
      DO 80 I=1,M
      SUM=0.0
      IF (I-2) 60,65,70
60    DO 61 J=1,M
61    SUM=SUM+F(J)
      GO TO 75
65    DO 66 J=1,M
66    SUM=SUM+R(J)*F(J)
      GO TO 75
70    V(1)=1.0
      DO 72 J=1,M
      V(2)=R(J)
      DO 71 K=3,I
71    V(K)=2.*R(J)*V(K-1)-V(K-2)
72    SUM=SUM+F(J)*V(I)
75    C(I)=2.0*SUM/ZM
80    CONTINUE
      C(1)=C(1)/2.
      ZIP=XMIN
      NN=N-1
      DO 93 J=1,NN
93    XMIN=XMIN+DELTX
      SUM1=(XMIN+ZIP)/2.0
      SUM2=(XMIN-ZIP)/2.0
      NN=M-1
```

```
C
C  ** CONVERT CHEBYSHEV SERIES TO ITS EQUIVALENT POWER SERIES
C
      F(1)=C(1)
      F(2)=C(2)
      IF (M.LE.2) GO TO 97
96    DO 98 K=1,M
      V(K)=0.0
      Y(K)=0.0
98    F(K+2)=0.0
      V(2)=1.0
      DO 97 K=3,M
      ZIP=K-1
      P=3.14159265*ZIP/[illegible]
      Y(1)=DCOS(P)
      DO 94 J=2,K
94    Y(J)=2.*V(J-1)-Y(J)
      DO 95 J=1,K
      F(J)=F(J)+C(K)*Y(J)
      ZIP=V(J)
      V(J)=Y(J)
95    Y(J)=ZIP
99    CONTINUE
C
C  ** GO BACK TO ORIGINAL INTERVAL
C
97    Y(1)=F(1)
      DO 81 K=2,M
81    Y(K)=0.0
      DO 83 K=2,M
      L=K-1
      Y(K)=Y(K)+F(K)/SUM2**L
      SUM3=1.0
      ZIP=1.0
      X2=K
      DO 82 J=1,L
      X1=J
      SUM3=SUM3*Y1
      DV=SUM3*SUM2**L
      ZIP=ZIP*(M2-Y1)
82    Y(K-J)=Y(K-J)+(ZIP*SUMI**J*(-1.)**J*F(K))/DV
83    CONTINUE
      RETURN
      END
C
C
C
      SUBROUTINE PATTRN
C
C  ** THIS SUBPROGRAM 'PATTRN' WILL FIND THE MINIMUM POINT IN THE
C     DESCENT DIRECTION BY THE FOLLOWING PRECEDURES.
C
C   1. FIND DESCENT DIRECTION FROM EXPLORATORY MOVE.
C   2. FIND MINIMUM POINT IN THAT DIRECTION USING FIBONACCI SEARCH.
C   3. CHECK THE CONVERGENCE CRITERION AND THE MAXIMUM NUMBER OF STEP
C      REDUCTIONS.
CC  4. IF NOT SATISFIED, ITERATE UNTIL THE CRITERION IS SATISFIED.
C
      IMPLICIT DOUBLE PRECISION*6 (A-H,O-Z)
      DIMENSION VAR(6),STEP(6) OVAR1(6) EXN(6) EXO(6),CH(II),CL(7)
      COMMON /COM1/NVAR,XJ/COM2/STEP/COM3/NVAR/COM4/C/PATN/MNRED,XMU
     1       /E PL/EXOBJ STOBJ/FIPO/EXO EXN ,LAMDA,OBJP
     2       /OBJE/R WT PNLT/OBJZ/OBJ/CTRL/NREDI
     3       /HFLD/CH/LOSS/CL/SELE/ISEC,PWR V NRPM,IP TORG
C
C  ** INITIALIZATION
C
      Y=0.0D0
      NRED=0
      EXOBJ=0.0D0
      CALL DESIGN(Y)
      OBJP=OBJ
      STOBJ=OBJ
      OBJX=OBJ
      IEPS=0
C
C  ** STORE THE INITIAL VALUES OF DESIGN VARIAPLES AS STARTING POINTS
C     OF PATTRN MOVE.
C
      DO IO I=1.NVAR
      EXO(I)=VAR(I)
10    CONTINUE
C
C  ** START EXPLORATORY MOVE
C
20    STOPJ=OPJF
30    CALL EXPLOR
      CALL OESIGN(Y)
C
C  ** CHECK IF THE CONSTRAINTS ARE SATISFIED.
C   * IF NOT SATISFIED. REDUCE THE STEP SIZE.
C
      IF (C.EG.I.ODO) GO TO 40
C   * IF SATISFIED. COMPARE THE OPJECTIVE FUNCTION.
C
      IF(EXOBJ.LT.OPJF) GO TO 60
40    NRED=NRED+1
C
C  ** IF THE NUMBER OF STEP REDUCTION IS GREATER THAN THE LIMIT.THEN STOP.
C   * IF NOT. CONTINUE STEP REDUCTION AND REPEAT EXPLORATORY MOVE.
C
C  ** START THE PATTRN MOVE ALONG THE DESCENT DIRECTION
C
      IF(NRED.GT.MNRED) GO TO I70
      DO 50 I-I.NVAR
      STEP(I)=STEP(I)*XMU
50    CONTINUE
      GO TO 30
C
```

```
60    WRITE (4,5) NRED,OBJP,EXOBJ,(VAR(I),I=1,NVAR)
5     FORMAT(' EXPLORATORY MOVE  NRED-',I3,3X,'OBJP=',D14
     I3X 'EXOBJ=',D14.6/6D17,9)
C
      OBJP=EXOBJ
      DO 70 I-1,NVAR
      EXN(I)=VAR(I)
      OVARI(I)=VAR(I)
70    CONTINUE
      CALL FIBONA
      WRITE (4,31) NRED,OBJP,OBJ,(VAR(I),I=1,NVAR)
31    FORMAT(' FIBONACCI MOVE  NRED=',I3,3X,'OBJP=',D14.6
     I3X,'OBJ=',D14.6/6D17,9)
C
C  ** CHECK THE CONVERGENCE CRITERIAN TWICE,
C   * IF SATISFIED OR THE MAXIMUM NUMBER OF STEP REDUCTION IS LARGER TH
C     THE LIMIT, STOP OPTIMIZATION,
C   * IF NOT, ITERATE OPTIMIZATION PROCESS AGAIN.
C
      STOP=DABS((OBJX-OBJ)/OBJ)
      IF (STOP.LT.0,1D-6) GO TO 90
      GO TO 80
90    IF (IEPS.EQ.I) GO TO 110
      IEPS-1
80    OBJX=OBJ
      OBJP=OBJ
      DO 100 I=I,NVAR
      OVARI(I)=VAR(I)
      EXD(I)-VAR(I)
100   CONTINUE
      NRED=NRED+I
      IF (NRED,GT,MNRED) GO TO 120
      GO TO 20
120   WRITE(4,95) NRED,MNRED
95    FORMAT(' NUMBER OF REDUCTION(',I3,') EXCEEDS',I6)
      GO TO 140
```

```
110   WRITE(4,115) STOP
115   FORMAT(' OPTIMIZATION CRITERION EPS=' 2X,F12.10)
140   WRITE(4,145)
145   FORMAT(' RESULT OF OPTIMIZATION')
      WRITE (4,150) OBJ,WT,(VAR(I),I=1,NVAR)
150   FORMAT(//' OBJ=',D16.8,3X,'WEIGHT=',D14.8//6D14.7///)
      NREDI=NRED
      NRED=0
      EXOBJ=0.0D0
      RETURN
      END
C
C
C
      SUBROUTINE EXPLOR
C
C  ** THIS SUBPROGRAM 'EXPLOR' WILL FIND THE DESCENT DIRECTION OF THE
C     OBJECTIVE FUNCTION.
C
      IMPLICIT DOUBLE PRECISION*8 (A-H,O-Z)
      DIMENSION VAR(6),STEP(6),VOLD(6)
      COMMON /COM1/VAR,XJ/COM2/STEP/COM3/NVAR/COM4/C
     1       /EXPL/EXOBJ,STOBJ/OBJ2/OBJ
     2       /HFLD/CH/LOSS/CL/SELE/ISEC,PWR,V,NRPM,IP,TORQ
C
C  ** INITIALIZATION
C
      Y=0.0D0
      EXOBJ=STOBJ
C
C  ** MOVE TO THE INCREASING DIRECTION OF DESIGN VARIABLES.
C
      DO 100 I=1,NVAR
      VOLD(I)=VAR(I)
      VAR(I)=VAR(I)+STEP(I)
      CALL DESIGN(Y)
      FIELD=FIELD+CH(I)*P**II
30    CONTINUE
      URUO=B/FIELD
40    RETURN
      END
C
C
C
      SUBROUTINE CORLOS (B,CLOS)
C
C  ** THIS SUBPROGRAM 'CORLOS' WILL COMPUTE THE CORE LOSS OF MAGNETIC
C     MATERIAL IN UNIT WEIGHT FROM THE INFORMATION OF MAGNETIC FLUX DENSITY.
      IMPLICIT DOUBLE PRECISION*8 (A-H,O-Z)
      COMMON /LOSS/CL(9)
C
      CLOS=0.0D0
      DO 10 I=1,9
      II=I-1
      CLOS=CLOS+CL(I)*B**II
10    CONTINUE
      RETURN
      END
C
C
C
      FUNCTION IF(M)
C
C  ** THIS SUBPROGRAM FUNCTION 'IF(M)' WILL GENERATE THE FIBONACCI NUMBER.
C
      INTEGER IX(50)
      IX(1)=1
      IX(2)=2
      M1=M-2
      IF (M1.LE.0) GO TO 30
      DO 10 I=1,M1
      IX(I+2)=IX(I+1)+IX(I)
```

```
10    CONTINUE
      IF=IX(M)
      GO TO 50
30    IF (MI.EQ.0) GO TO 40
      IF=1
      GO TO 50
      IF=1
40    IF=2
50    RETURN
      END
C
C
C
      SUBROUTINE MOVE (D)
C
C  ** THIS SUBPROGRAM 'MOVE' WILL COMPUTE THE NEW VALUES OF DESIGN
C     PARAMETERS MOVED FROM OLD VALUES.
C
      IMPLICIT DOUBLE PRECISION*6 (A-H,O-Z)
      DIMENSION VAR(6),EXO(6),EXN(6)
      COMMON /COM1/VAR,KJ/COM3/NVAR/FIFO/EXO,EXN,XLAMDA,OFJP
      DO 10 I=1,NVAR
      VAR(I)=EXN(I)*(D+1)-D*EXO(I)
10    CONTINUE
      RETURN
      END
C
C
C
      SUBROUTINE FIBONA
C
C  ** THIS SUBPROGRAM 'FIBONA' WILL DETERMINE THE MINIMUM OF OBJECTIVE
C     FUNCTION IN THE DESCENT DIRECTION USING THE FIBONACCI SEARCH
C
      IMPLICIT DOUBLE PRECISION*6 (A-H,O-Z)
      DIMENSION VAR(6),EXD(6),EXN(6),A(20),B(20),XL(20),XM(20)
      COMMON /COM1/VAR,XJ/COM3/NVAR/COM4/C/FIFO/EXD,EXN,XLAMDA,OBJP
     I       /OBJ2/OBJ/HFLD/CH/LOSS/CL/SELE/ISEC,FWR,V,NRPM,IP,TORQ
C
C  ** INITIALIZATION
C
      Y=0.0D0
      N=10
      A(I)=-I.0D0
      C=0.0D0
      D=0.0D0
      G=1.0D0
      NN=0
      OBJPI=OBJP
C
C  ** FIND ANOTHER END OF SEARCH REGION.
C
10    D=D+G
      CALL MOVE(D)
      CALL DESIGN(Y)
C
C  ** CHECK IF THIS MOVEMENT SATISFIES THE CONSTRAINT.
C   * IF SATISFIED  COMPARE THE OBJECTIVE FUNCTION
C   * IF NOT, REDUCE THE MOVEMENT SIZE
C
      IF (C.NE.1.0D0) GO TO 20
      D=D-G
      G=G*0.5D0
      GO TO 10
20    IF (OBJ.GE.OBJPI) GO TO 30
      OBJPI=OBJ
      IF (NN.GT.50) GO TO 200
      NN=NN+I
      GO TO 10
30    B(1)=D
C
```

```
      N2=N-2
      N1=N-1
C
C  ** GENERATE THE FIRST REGION OF FIBONACCI SEARCH.
C
      XL(1)=A(1)+(B(1)-A(1))*IF(N2)/IF(N)
      XM(1)=A(1)+(B(1)-A(1))*IF(N1)/IF(N)
      CALL MOVE(XL(1))
      CALL DESIGN(Y)
      OBJL=OBJ
      CALL MOVE(XM(1))
      CALL DESIGN(Y)
      OBJM=OBJ
C
C  ** COMPARE THE OBJECTIVE OF TWO POINTS AND LARGER OBJECTIVE FUNCTION
C     POINT WILL BE DISCARDED.
C   * IF OBJL > OBJM, LEFT RANGE WILL BE DISCARDED, OTHERWISE RIGHT REGION
C     WILL BE DISCARDED.
C
      K=1
C
40    IF (OBJL.GT.OBJM) GO TO 50
      A(K+1)=A(K)
      B(K+1)=XM(K)
      XM(K+1)=XL(K)
      OBJM=OBJL
      XL(K+1)=A(K+1)+(B(K+1)-A(K+1))*IF(N-K-2)/IF(N-K)
      IF (K.EQ.(N-2)) GO TO 60
      CALL MOVE(XL(K+1))
      CALL DESIGN(Y)
      OBJL=OBJ
      K=K+1
      GO TO 40
50    A(K+1)=XL(K)
      B(K+1)=B(K)
      XL(K+1)=XM(K)
      OBJL=OBJM
      XM(K+1)=A(K+1)+(B(K+1)-A(K+1))*IF(N-K-1)/IF(N-K)
      IF (K.EQ.(N-2)) GO TO 70
      CALL MOVE(XM(K+1))
      CALL DESIGN(Y)
      OBJM=OBJ
      K=K+1
      GO TO 40
C
C  ** REDUCE THE REGION OF UNCERTAINTY FOR THE LAST TWO STEPS OF
C     COMPARISON.
C
70    CALL MOVE(XM(K+1))
      CALL DESIGN(Y)
      OBJM=OBJ
      GO TO 90
60    CALL MOVE(XL(K+1))
      CALL DESIGN(Y)
      OBJM=OBJ
80    IF (OBJL.GE.OBJM) GO TO 90
      OBJ1=OBJL
      XLMD=XL(K+1)
      XLMD1=(A(K+1)+XM(K+1))/2.0
      GO TO 100
90    OBJ1=OBJM
      XLMD=XM(K+1)
      XLMD1=(XL(K+1)+B(K+1))/2.0
C
C  ** DECIDE THE LAST RESULT OF FIBONACCI MOVE
C
100   CALL MOVE(XLMD1)
      CALL DESIGN(Y)
      OBJF=OBJ
      IF (OBJ1.LT.OBJF) GO TO 110
      OBJ=OBJF
      XLAMDA=XLMD1
      GO TO 200
110   OPJ=OPJI
      XLANDA=XLMD
200   NP=K+1
      RETURN
      END
C
C
C
      SUBROUTINE DESIGN(Y)
C
C  ** THIS SUBPROGRAM 'DESIGN' WILL COMPUTE THE OBJECTIVE FUNCTION BY THE
C     FOLLOWING PROCEDURES.
C
C   A. COMPUTE MAGNETIC FLUX
C   B. CHECK CONSTRAINTS
C     * MAGNETIC
C     * ELECTRICAL
C     * THERMAL
C     * GEOMETRICAL
C   C. COMPUTE THE OBJECTIVE FUNCTION
C
      IMPLICIT DOUBLE PRECISION*6 (A-H,O-Y)
      DIMENSION VAR(6),CNTR(8),CH(11),CL(9)
C
      COMMON /COM1/VAR,XJ/COM3/NVAR/COM4/C/OBJE/R,WT,PNLT
     1       /OPJ2/OBJ/CTRL/NRED1/OUTP/J
     2       /HFLD/CH/LOSS/CL/SELE/ISEC,PWR,V,NRPM,IP,TORQ
     3       /THER/TLOSS,ARE,GA,CA,CF,HC,AA2/RESL/TEMP,NADJ
C
C  ** CHECK THE NON-NEGATIVITY OF DESIGN PARAMETERS
C     IF VARIABLES ARE EQUAL OR LESS THAN ZERO THE DESIGN BECOMES NOT
C     FEASIBLE FOR GEOMETRICAL REASONS
C
      C=0.0D0
      DO 10 I=1,NVAR
      IF (VAR(I).GT.1.0D-7) GO TO 10
      C=1.0D0
10    CONTINUE
      IF (C.EQ.1.0D0) GO TO 1000
C
C  ** THE VALUES OF GIVEN PARAMETERS ARE AS FOLLOWS.
C
C   * MECHANICAL PARAMETERS
C
      XKSA=0.45D0
      DSH=2.54D-2
      S-0.254D-2
      B=0.75*2.54D-2
      SR=3.0D0
C
C   * DENSITY OF MATERIAL
C
      DCF=8900.0D0
      DIN-1200.0D0
      DIR=7800.0D0
      DPM-7800.0D0
C
C   * ELECTRICAL & MAGNETIC PARAMETERS
C
      RHO=0.021D-6
      BR=0.85D0
      BS=1.8D0
      U0=12.566D-7
      RU=1.0D0
      A-2.0D0
      NAS=10
C
C   * THERMAL PARAMETERS
C
      CFE=420.0D0
      XKC=0.026D0
      VIS=1.849D-5
      DPR=1.1774D0
      F=0.715
      PR=0.72
      TMAX=140.0D0
C
C     * CHANGE GUESS(I) INTO ACTUAL VARIABLES.
C
      D=VAR(1)
      XL=VAR(2)
      AA=VAR(3)
      YLM=VAR(4)
      TF=VAR(5)
C
      G=(0.3175-25.83/(D/2.54D0+9.0D0))/100.0D0
      X=3.14159265D0/4.0D0
      PY=3.14159265D0
      W=PY*NRPM/30.0D0
      ALPHA=PY/6.0D0
      M=IP/2
C
C  ** 1. IF THIS DC MOTOR IS FOR THE GENERAL PURPOSE.THEN COMPUTE THE
C        DIAMETER OF ARMATURE WINDING.
C     2. IF THIS DC MOTOR IS FOR THE MULTI-LINK MANIPULATOR THEN SKIP
C        THESE PROCEDURES.
C
15    WAS=(AA/(XKSA*SR))**0.5D0
      DAS=SR*WAS
      XLA=(PY*(D-DAS)/(IP*DCOS(ALPHA))+2.0D0*B+DAS+YL)*2.0D0
      XI2R=XJ**2.0D0*RHO*XLA*AA*NAS*M
      IF (ISEC.EQ.2) GO TO 19
      TORQ=POR/W
      DWA=((PWR+XI2R)/(V*XJ*A*X))**0.5D0
      XN7=AA/(X*DWA**2.0D0)
      ZN1=SNGL(XN7)
      IF((ZNI-IFIX(ZNI)).GE.0.5D0) NI=IFIX(ZNI)+1
      IF((ZNI-IFIX(ZNI)).LT.0.5D0) NI=IFIX(ZNI)
      AAI=X*DWA**2.0D0*NI
      IF(DABS(AA-AAI).LT.0.1D-5) GO TO 25
      AA=AAI
      GO TO 15
C
C  ** COMPUTE ARMATURE RESISTANCE & INDUCED VOLTAGE
C
25    N=NI*NAS*M
      XIA=XJ*Y*DWA**2.0D0*A
      AR=YLA*N*RHO/(A**2.0D0*X*DWA**2.0D0)
      EG=V-XIA*AR
      IF (EG.GE.0.1D-6) GO TO 27
      C=1.0D0
      GO TO 1000
C
C  ** COMPUTE MAGNETIC FLUX OF GENERAL PURPOSE MOTOR.
C
27    FLUX=PY*A*EG/(N*IP*W)
      GO TO 29
C
C  ** COMPUTE MAGNETIC FLUX OF MOTOR FOR MULTI-LINK MANIPULATOR.
C
19    FLUX=2.0D0*PY*TORQ/(IP**2*NAS*AA*XJ)
C
C  ** COMPUTE THE THICKNESS OF PERMANENT MAGNET
C
C   * MAGNETIC FLUX DENSITY OF EACH PART
C
29    TTH=(PY*D-IP*NAS*WAS)/(IP*NAS)
      TTH3=(PY*(D-4.0D0/3.0D0*DAS)-IP*NAS*WAS)/(IP*NAS)
      XK=1.0D0/(1.0D0+WAS/(5.0D0*G))
      GE=G*(TTH+WAS)/(TTH+WAS*XK)
      AM=XLM*XL
      AG=AM
      AT=TTH3*XL*NAS*XLM/(PY*(D+GE)/IP)
      AFF=(D-2.0D0*DAS-DSH)/2.0D0
      AAY=XL*AFF
      AFY=TF*XL
      PM=FLUX/AM
      PG=PM
      BT=FLUX/AT
      BAY=FLUX/(2.0D0*AAY)
      BFY=FLUX/(2.0D0*AFY)
C
C   * CONSTRAINT CHECK
C
      CNTR(1)=NAS/(PY*(D-2.0D0*DAS)/(IP*NAS))-I.0D0
      CNTR(2)=(2.0D0*DAS+DSH)/D-1.0D0
      CNTR(3)=BT/BS-I.0D0
      CNTR(4)=BAY/BS-I.0D0
      IF (IP.EQ.2) CNTR(5)=XLM/(PY*(D+G)/3.0D0)-1.0D0
      IF (IP.GT.2) CNTR(5)=XLM/(0.3D0*PY*(D+G)/IP)-I.0D0
      CNTR(6)=BFY/BS-I.0D0
      DO 60 I=I,6
      IF (CNTR(I).LT.0.0D0) GO TO 60
      C=I.0D0
60    CONTINUE
      IF (C.EQ.I.0D0) GO TO I000
C
C   * COMPUTE THE RELUCTANCE OF EACH PART INSIDE THE PM MOTOR
C
      RG=GE/(UO*AG)
      CALL HFIELD(BT,URUO)
      RT=DAS/(URUO*AT)
      CALL HFIELD(BAY,URUO)
      PAY=(D-2.0D0*DAS+DSH)*X
      RAY=PAY/(URUO*AAY)
      CALL HFIELD(BFY,URUO)
      F)=FLUX*(2.0D0*(RG+RT)+0.5D0*RAY)
      F'=FLUX*(2.0D0/(RU*UO*AM)+PY/(2.000*URUO*AFY))
      F3=FLUX*PY*(D+2.0D0*CE+TF)/(4.0D0*URUO*AFY)
      CNTR(7)=F2*RU*UO/(2.0D0*PR)-1.0D0
      IF (CNTR(7).LT.0.0) GO TO 70
      C=1.0D0
      GO TO 1000
C
C   * COMPUTE THE THICKNESS OF PERMANENT MAGNETIC.
C
70    TM=1.3D0*(F1+F3)/(2.0D0*PR/(RU*UO)-F2)
      PFY=(D+2.0D0*G+2.0D0*TM+TF)*PY/2.0D0
      RFY=PFY/(URUO*AFY)
      RM=TM/(UO*RU*AM)
C
C  ** COMPUTE LOSS OF MOTOR
C
      CALL CORLOS(FAY,CLDS)
      WAY=DIR*XL*PI*(D+2.0D0*DAS)**2.0D0
      CLOSS1=CLOS*WAY/0.45D0
      CALL CORLOS(BT,CLDS)
      WAT=X*DIR*D**2.0D0*XL-WAY-IP*NAS*DIR*DAS*WAS*XL
      CLOSS2=CLDS*WAT/0.45D0
      CLOSS=(CLOSS1+CLOSS2)*2.5D0
      XLMECH=(XI2R+CLOSS)*0.1D0
      TLOSS=CLOSS+XI2R+XLMECH
C
C  ** COMPUTE THE OBJECTIVE FUNCTION
C
C   * WEIGHT OF ARMATURE PART
C
321   WAW=DCP*XLA*AA*NAS*M
      WAI=DIN*XL*(DAS*WAS-AA)*IP*NAS
      WA=WAW+WAI+WAY+WAT
C
C   * WEIGHT OF FIELD PART
      WPM=XLM/(PI*(D+2.0D0*G))*X*((D+2.0D0*G+2.0D0*TM)
     1**2.0D0-(D+2.0D0*G)**2.0D0)*XL
```

```
      WFP=2.0D0*DPM*VPM
      WFY=D*PI*YL*X*((D+2.0D0*G+2.0D0*TM+2.0D0*TF)**2.0D0-
     1(D+2.0D0*G+2.0D0*TM)**2.0D0)
      WF=WFP+WFY
C
C   * TOTAL WEIGHT
      WT=WA+WF
C
C  ** CHECK THE TEMPERATURE CONSTRAINT
C
      IF (Y.EQ.4.0D0) GO TO 44
      GO TO 77
44    SF=D*NRPM/60.0D0*PY
C
      DD=D+2.0D0*(TM+TF+G)
      ARE=PY*DD*YL+0.5D0*PY*DD**2.0D0
      AOZ=PY*D*YL+0.5D0*PY*D**2.0D0
      GA=2.0D0*XKC*YL/(D+2.0D0*G)*(IF*XLM/DLOG((D+2.0D0*G)/D)+
     1(PY*(D+2.0D0*G)-IF*XLM)/DLOG((D+2.0D0*G+TM)/D))
      YNU=VIS/DAR
      RE=G*SF/YNU
      CFR=(10.0D0/RE)**0.67D0
      XNUS=F*PR*RE*(CFR/2.0D0)**0.5D0/10.0D0/(PR+DLOG(1.0D0+5.0D0*PR)+
     10.5D0*DLOG(RE/120.0*(CFR/7.0)**0.5D0))
      HG=XKC*XNUS/(2.0D0*G)
      CA=WA*CFE
      CF=WF*CFE
C
C  ** NOW COMPUTE THE TEMPRATURE RISE & CHECK THERMAL CONSTRAINT
C
      CALL THERML
      GO TO 1000
C
C   * BARRIER FUNCTION
C
```

```
77    BAR11=-(1.0D0/CNTR(1)+1.0D0/CNTR(2)+1.0D0/CNTR(3)+1.0D0/CNTR(4)
     1+1.0D0/CNTR(5)+1.0D0/CNTR(6)+1.0D0/CNTR(7))
      BAR1=BAR11*R
C
C  ** OBJECTIVE FUNCTION
C
      WTNG=1.0D0
      OBJ=WT*WTNG+(1.0D0-WTNG)*TLOSS+BAR1
      PNLT=BAR1
      IF (Y.GE.1.0D0) GO TO 100
      GO TO 1000
100   IF (ISEC.EQ.1) GO TO 186
      W=NRPM*PY/30.0D0
      PWR=TORQ*W
      DWA=((PWR*X12R)/(V*XJ*A*X))**0.5D0
      XN7=AA/(K*DWA**2.0D0)
      ZN1=SNGL(XN7)
      N1=IFIX(ZN1)
      N=N1*NAS*M
      XIA=XJ*X*DWA**2.0D0*A
      AR=XLA*N*RHO/(A**2.0D0*K*DWA**2.0D0)
      EG=V-XIA*AR
      EFF=PWR/(PWR+TLOSS)*100.0D0
      PWR1=PWR/746.0D0
      GO TO 188
186   EFF=PWR/(PWR+TLOSS)*100.0D0
      PWR1=PWR/746.0D0
188   WRITE(7,260) J,R,NRED1,(VAR(I),I=1,5),TM,DWA,TYR
260   FORMAT('0',I2,3X,D7.1,3X,I4,5X,8(D10.4,2X))
      WRITE(7,270) OBJ,WT,EFF,EG,FLUX,PM,PT,PAY,BFY
270   FORMAT(1X,D15.7,2X,D15.8,7(2X,D10.4))
      IF (Y.EQ.7.0D0) GO TO 300
      GO TO 1000
300   YY=PY*D/IP
      XLA=PY*(N/A)**2*PL*(XLM/ \)**3 [illegible]
      D=D*100.0D0
      XL=XL*1000.0D0
      XLPERD=XL/D
      AA=AA*1.0D4
      XLM=XLM*100.0D0
      TF=TF*100.0D0
      XJ=XJ/10000.0D0
      XINERT=WA*(D/2.0D0)**2.0D0/[illegible]
      ACCEL=TORQ/XINERT
      XKE=EQ/W
      XKT=TORQ/XIA
      DWA=DWA*100.0
      ET=YLA/AR*1000.0D0
      YLA=YLA*1000.0D0
      XMT=AR*XINERT/(XKE*XKT)*1000.0
C
C  ** PRINT THE RESULT OF OPTIMIZATION
C
      WRITE (2,101)
101   FORMAT('1', RESULT OF OPTI[illegible]
      WRITE(2,102)
102   FORMAT('}  INPUT DATA',/)
      IF (ISEC.EQ.1) GO TO 222
      WRITE (2,107) TORQ
107   FORMAT(5X,'TORQUE (N-M)...............................' F8.3)
      WRITE (2,401) IP
401   FORMAT(5X,'NUBER OF POLES.............................' I4)
      WRITE (2,402) NRPM
402   FORMAT(5X,'MAXIMUM SPEED (RPM)........................',I4)
      WRITE (2,403) PWR1
403   FORMAT(5X,'POWER(HP)..................................',F7.2,/
      GO TO 273
222   WRITE(2,301) PWR1
301   FORMAT(5X,'POWER (HP)..................................',F5.[illegible]
      WRITE(2,302) V
302   FORMAT(5X,'VOLTAGE (VOLT)..............................',F5.0)
      WRITE(2,303) XIA
```

```
303   FORMAT(5X,'CURRENT (AMP)...................'.F6.[illegible]
      WRITE(2,304) NRPM
304   FORMAT(5X,'SPEED (RPM).......................'.I4)
      WRITE (2,305) IP
305   FORMAT(5X,'NUMPER OF POLES...................'.I4,/)

273   WRITE(2,108)
108   FORMAT('2. DESIGN PARAMETERS',/)
      WRITE (2,109) D
109   FORMAT(5X,'ROTOR DIAMETER (CM)..............'.F8.3)
      WRITE (2,110) YL
110   FORMAT(5X,'ROTOR LENGTH (CM)................'.F8.3)
      WRITE (2,111) AA
111   FORMAT(5X,'SLOT AREA (CM**2)................'.F8.3)
      WRITE (2,112) WLM
112   FORMAT(5X,'MAGNET WIDTH (CM)................'.F8.3)
      WRITE (2,113) TF
113   FORMAT(5X,'STATOR THICKNESS (CM)............'.F8.3)
      WRITE (2,441) XJ
441   FORMAT(5X,'CURRENT DENSITY (AMP/CM**2)......'.F8.3)
      WRITE (2,114) YLPERD
114   FORMAT(5X,'RATIO (L/D)......................'.F8.3,/)
      WRITE (2,115)
115   FORMAT('3. ELECTRICAL SPECIFICATIONS',/)
      WRITE(2,116) N1
116   FORMAT(5X,'NUMPER OF TURNS PER SLOT.........'.I4)
      WRITE(2,117) NAS
117   FORMAT(5X,'NUPER OF SLOTS PER POLE..........'.I4)
      WRITE (2,118) N
118   FORMAT(5X,'TOTAL NUMPER OF TURNS............'.I4)
      WRITE (2,120) DWA
120   FORMAT(5X,'DIAMETER OF ARMATURE COIL (CM)...'.F8.3)
      WRITE (2,121) AR
121   FORMAT(5X,'RESISTANCE OF ARMATURE COIL (OHM).....'.F8.3)
      WRITE (2,122) YLA
122   FORMAT(5X,'INDUCTANCE OF ARMATURE COIL (MH).......'.D10.3)
```

```
      WRITE (2.123) EG
123   FORMAT(5X.'INDUCED VOLTAGE (VOLT)...............................',F8.3)
      WRITE (2.124)
124   FORMAT('4. MAGNETIC SPECIFICATIONS'./)
      WRITE (2.125) FLUX
125   FORMAT(5X.'FLUX (WB)........................................',F8.3)
      WRITE (2.126)
126   FORMAT(5X.'FLUX DENSITY (TESLA)')
      WRITE (2.127) BM
127   FORMAT(7X.'MAGNET...................................',F8.3)
      WRITF(2.128) BG
128   FORMAT(7X.'AIRGAP...................................',F8.3)
      WRITE (2.129) BT
129   FORMAT(7X.'ARMATURE TEETH ..........................',F8.3)
      WRITE (2.130) BAY
130   FORMAT(7X.'ARMATURE YOKE............................',F8.3)
      WRITE (2.131) BFY
131   FORMAT(7X.'FIELD YOKE...............................',F8.3 /)
      WRITE (2.132)
132   FORMAT('5. LOSS (WATT)'./)
      WRITE (2.133) XI2R
133   FORMAT(5X.'COPPER LOSS ...............................',F8.3)
      WRITE (2.134) CLOSS
134   FORMAT(5X.'CORE LOSS .................................',F8.3)
      WRITE (2.137) XLMECH
137   FORMAT(5X.'MECHANICAL LOSS ...........................',F8.3./)
      WRITE (2.138) TLOSS
138   FORMAT(5X.'TOTAL LOSS.................................',F8.3./)
      WRITE (2.139) EFF
139   FORMAT(5X.'EFFICIENCY (%).............................',F8.3./)
      WRITE (2.135)
135   FORMAT('6. WEIGHT (KG)'./)
      WRITE (2.136) WA
136   FORMAT(5X.'ARMATURE (ROTOR)...........................',F8.3)
```

```
      WRITE (2,140) WAW
140   FORMAT(7X,'WINDING...............................',F8.3)
      WRITE (2,141) WAY
141   FORMAT(7X,'YOKE..................................',F8.3)
      WRITE (2,142) WAT
142   FORMAT(7X,'TEETH.................................',F8.3)
      WRITE (2,143) WF
143   FORMAT(5X,'FIELD (STATOR)..........................',F8.3)
      WRITE (2,144) WFP
144   FORMAT(7X,'MAGNET................................',F8.3)
      WRITE (2,145) WFY
145   FORMAT(7X,'YOKE..................................',F8.3 /)
      WRITE (2,146) WT
146   FORMAT(5X,'TOTAL WEIGHT............................',F8.3 /)
      WRITE (2,147) ACCEL
147   FORMAT('7, ACCELERATION (RAD/SEC**2)...............',F8.3,/)
      WRITE (2,148) TORQ
148   FORMAT(5X,'TORQUE (N-M)............................',F8.3)
      WRITE (2,149) XINERT
149   FORMAT(5X,'INERTIA (KG-M**2)........................',F7.5 /)
      WRITE (2,150)
150   FORMAT('8, PERFORMANCE CONSTANT',/)
      WRITE (2,151) XKE
151   FORMAT(5X,'VOLTAGE CONSTANT (KE:V.SEC/RAD)..........',F8.3)
      WRITE (2,152) XKT
152   FORMAT(5X,'TORQUE CONSTANT (KT,N-M/A)...............',F8.3)
      WRITE (2,153) ET
153   FORMAT(5X,'ELECTRICAL TIME CONSTANT (MS)............',D10.3)
      WRITE (2,154) XMT
154   FORMAT(5X,'MECHANICAL TIME CONSTANT (MS)............',F8.3,/)
      WRITE (2,155) TEMP
155   FORMAT('9, ARMATURE TEMPERATURE RISE................',F7.2)
1000  RETURN
      END
C
C
```

```
      SUBROUTINE THERML
C
C  ** THIS SUBPROGRAM 'THERML' WILL COMPUTE THE TEMPERATURE RISE USING
C     THE NUMERICAL MMETHOD 'PREDICTOE-CORRECTOR METHOD
C
      IMPLICIT DOUBLE PRECISION*8 (A-H,O-Y)
      DIMENSION YP(4),YC(4)
      COMMON /COMM/X(50),Y(3,50),FFI(3,50)
     1        /THER/PL,A,DA,CA,CF,HO AA/RESL/TEMP,NADJ
C
C  ** INITIALIZATION
C
      EPS-0.001D0
      DELT-100 0D0
      N=4
      NO=2
      YMIN-12,0D0
      XMAX-3600.0D0
      TER-25.0D0
5     X(I)=0,0D0
      Y(I,1)=0 0D0
      Y(2,I)=0,0D0
C
      CALL RKUTTA(DELT,N,NO)
      J=0
10    NPJ=N+J
      DO 15 I=1,NO
15    YP(I)=Y(I,NPJ)+DELT*(55.0*FF1(I NPJ)-59.0*FF1(I,NPJ-1)+
     &37.0*FFI(I,NPJ-2)-9,0*FF1(I NPJ-3))/24 0
      H=0
      X(NPJ+1)=X(NPJ)+DELT
20    DO 25 I=1,NO
25    FFI(I,NPJ+1)-F(I,X(NPJ+I),YP(1),YP(2),YP(3),YP(4))
      DO 30 I-1,NO
30    YC(I)=Y(I,NPJ)+DELT*(9,0*FF1(I,NPJ+1)+19.0*FF1(I,NPJ)-
     &5.0*FFI(I,NPJ-1)+FFI(I NPJ-2))/24 0
```

```
      DELY=ABS((YC(1)-YF(1))/YC(1))
      IF (DELY-EPS) 40,40,55
40    DO 45 I=1,NO
45    Y(I,NPJ+1)=YC(I)
      IF(X(NPJ+1).GE.XMAX) GO TO 96
      J=J+1
      GO TO 10
55    IF(M.GT.3) GO TO 70
      DO 65 I=1,NO
65    YF(I)=YC(I)
      M=M+1
      GO TO 20
70    WRITE(9,71)
71    FORMAT(//,5X,' 3 ITERATIONS ARE NOT SUFFICI[illegible]
      DELT=DELT*0.5
      IF(DELT.LT.XMIN) GO TO 80
      GO TO 5
80    WRITE(9,81)
81    FORMAT(//,' SOMETHING WAS WRONG IN THERMAL ANALYSIS')
96    TEMP=Y(1,NPJ+I)
      RETURN
      END
C
C
C
      SUBROUTINE RKUTTA(DELT,N,NO)
      IMPLICIT DOUBLE PRECISION*8 (A-H,O-Z)
      DIMENSION FF2(4),FF3(4),FF1(1)[illegible]
      COMMON /COMM/ (50),X(3,50),FF1[illegible]
     1       /THER/PL,A,CA,CA,CF[illegible]
C
      DO 10 J=1,N
      DO 20 I=1,NO
      FF1(I,J)=F(I,X(J),Y(1,J)[illegible]
20    Z(I)=Y(I,J)+DELT*FF1(I)[illegible]
      [illegible](J)+DELT/2.0
C
      DO 30 I=1,NO
      FF2(I)=F(I,XA,Z(1),Z(2),Z(3),Z(4))
30    Z(I)=Y(I,J)+DELT*FF2(I)
C
      DO 40 I=1,NO
      FF3(I)=F(I,XA,Z(1),Z(2),Z(3),Z(4))
40    Z(I)=Y(I,J)+DELT*FF3(I)
      X(J+1)=X(J)+DELT
      XA=X(J+1)
C
      DO 50 I=1,NO
50    FF4(I)=F(I,XA,Z(1),Z(2),Z(3),Z(4))
      DO 60 I=1,NO
60    Y(I,J+1)=Y(I,J)+DELY*(FF1(I,J)+2.0D0*(FF2(I)+FF3(I))+FF4(I))/6.0
10    CONTINUE
      RETURN
      END
C
C
C
      FUNCTION F(I,X,Y1,Y2,Y3,Y4)
C
C  ** THIS SUBPROGRAM FUNCTION 'F(I,X,Y1,Y2,Y3,Y4)' COMPUTES THE TEMPERATURE
C     RISE OF MOTOR.
C
      IMPLICIT DOUBLE PRECISION*6 (A-H,O-Y)
      COMMON /THER/FL,A,GA,CA,CF,HG,AA
      TER=25.0
      EMI=0.75
      TEA=Y1+TER
      TEF=Y2+TER
      TFI=(TEF+TER)/2.0+273.0
      DI=3.474*A
      PC1=(14.646*ABS(Y2)**0.25/(D)**0.25*ABS(TFI)**0.37)+9.072E-4*
     1ABS(TFI)**0.743/DI)*A*Y2
```

```
      PC2=5,71E-8*EM1*((TEF+273.0)**4-(TER+273.0)**4)*A
      PC=PC1+PC2
      PA1=GA*(TEA-TEF)
      PA1=0,0
      PA2=HG*AA*(TEA-TEF)
      PA3=5.71E-8*EMI*((TEA+273,0)**4-(TEF+273,0)**4)*AA
      PA=PA1+PA2+PA3
      VA=0.0
      PF=1280,0*VA*((TEA+TEF)/2,0-TER)
      IF (I.EQ.1) F=(PL-PA-PF)/CA
      IF (I.EQ.2) F=(PA-PC)/CF
      WRITE(9,1) PA,PC,F,PL,TEA,TEF
1     FORMAT('PA-TEF',6D15.5)
      RETURN
      END
```

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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